Curve Builder in DRIP

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Introduction

Framework Glossary

1. <u>Self-Jacobian</u>: Self-Jacobian refers to the Jacobian of the Objective Function at any point in the variate to the Objective Function at the segment nodes, i.e., $\frac{\partial Y(t)}{\partial Y(t_K)}$.
2. <u>Point-Measure State-Transform</u>: Point-Measure transform refers to the one-to-one transform between a state measure at a predictor ordinate and its corresponding observation (e.g., discount factor from zero-coupon bond price observations).
3. <u>Convolved-Measure State-Transform</u>: Convolved-Measure transform refers to the many-to-one transform between a state metric/predictor ordinate combination to a given observation, i.e., a set of state metric/predictor ordinate pairs together imply an observation (e.g., zero rates from swap fair premia).
4. <u>Discount-Curve Native Forward Curve</u>: For discount curves built out of instruments dependent on forward rates, those rates and their discount curve usage ranges together constitute the discount curve's native forward curve range.

Overview

1. <u>Smoothness Criterion Evolution</u>: Smoothness formulation is related to the minimization of strain energy (Schwarz (1989)), and the relation to Natural cubic spline (Burden and Faires (1997)), financial cubic spline (Adams (2001)) has been explored.
2. <u>Empirical vs. Theoretical Curve Builder Frameworks</u>: Zangari (1997) and Lin (2002) discuss this in detail.
   - Theoretical Term Structure posit explicit term structure for a variable known as short rate of interest whose values are extracted, possibly, from a statistical

analysis of market variables (Vasicek (1977), Cox, Ingersall, and Ross (1985), Rebonato (1998), Barzanti and Corradi (1998), Golub and Tilman (2000)).

- For bonds/treasuries see Nelson and Siegel (1987), Diament (1993), Svensson (1994), Soderlind and Svensson (1997), Tanggaard (1997). Effectiveness of such treatments is examined in Christensen, Diebold, and Rudebusch (2007), and Coroneo, Nyholm, and Vidova-Koleva (2008).
- Hybrid methods use empirically determined yield curve inside of a theoretical model (Hull and White (1990), Heath, Jarrow, and Morton (1990), Ron (2000)).
- A complete description of yield curve construction is given in Andersen and Piterbarg (2010).

Document Layout

Desired Curve Builder Features

Discount Curves

1. Exact instrument quote match: Does the builder scheme successfully construct the curve if the quotes do not pose arbitrage? Conversely, for inexact matches, does the builder algorithm converge rapidly, and minimal error (Hagan and West (2006), Hagan and West (2008))?
2. Implied Forward Rates: Taken to be typically 1m or 3m forwards – how smooth/positive/continuous are they (McCulloch and Kochin (2000))?
3. Locality: How local is the interpolating builder? If an input is changed, does the interpolator change only nearby, or is there spillover to non-adjacent far-off segments?
4. Stability of the Forward Rates: How sensitive are the forward rates to change in the inputs? The Jacobian analysis below shows the results for several splining scenarios.
    a. Forward rates are chosen for the curve behavior examination because it is the most elemental entity whose continuous/smooth behavior is meaningful to the practitioner.
5. Hedge Locality: Does most of the delta risk for a given instrument get assigned to the hedging instruments that have maturities close to the given instrument?
6. Sequential vs. Tenor Delta: Does the cumulative tenor delta equal to the aggregate (i.e., parallel shifted) delta? Le Floc'h (2013) examines the importance of this.

Curve Construction Methodology

Base Methodology

1. Instrument Setup: Construct the calibration instruments, and set up the instrument baseline. This includes initializing the span/segments, as well as the "tuning parameter" to achieve the desired "inner" and the "outer" calibrations.
2. Span/segment stretch set up: Calibrate the segments one by one using the calibration measures/inputs.
3. Tuning Adjustment: Adjust tuners to achieve the desired "boundary" condition.

State Span Design Components

1. Base Quantification Metric Retrieval: This refers to the functionality for retrieval of the State Quantification Metric Response Value at different predictor ordinates, the relative values, and canonical (possibly categorical) representations.
2. Targeted State Metric Computation: This functionality computes state/model specific targeted state metrics (e.g., LIBOR for a discount Curve, I Spread etc) that may be absolute or relative.
3. Sensitivity Jacobian: This functionality provides for the ability to extract sensitivity Jacobian at the following levels:
   - Cross Quantification Metric (Quantification Metric 1 to Quantification Metric 2) Sensitivity Jacobian
   - External Manifest Metric to Quantification Metric Sensitivity Jacobian
4. Calibration Input Manifest Measure Retrieval: This functionality records and retrieves the calibration input manifest measure set and other relevant calibration details.

- It needs to be remembered that the calibration input manifest measure set need not just be instrument quotes, but also “event” rates such as user specified turns meant to account for items such as year-end yield adjustments, periods of high activity etc: (Ametrano and Bianchetti (2009), Kinlay and Bai (2009)). In the case of turns, they may be modeled as discrete latent state jumps across specific pairs of dates, of a user-specified magnitude.
- Exogenously specified State Differentials => As just noted, certain state attributes maybe exogenously specified (e.g., turns, bases, etc:). These state shift differentials may be applied before or after the calibration step.

5. <u>Scenario State Span Re-construction</u>: This functionality re-constructs the state using adjusted, bumped, or otherwise scenario-tweaked quantification metrics and/or manifest measures.
6. <u>Boot State Span</u>: This functionality is used in boot state spans. Here, there needs to be the ability to set the boot values at the node knots, and the build the segment.
7. <u>Non-linear State Span</u>: This functionality sets up the non-linear fixed-point extraction process and the corresponding target match criterion evaluator.

Curve Calibration From Instruments/Quotes

1. <u>Construction from Single Instrument/Quote Set</u>: If there is only one type instrument/quote set to be calibrated from, you can simply “spline” through the constituent segments. In particular, if there are no value limitations/constraints, then spline construction may be achieved directly from the points (e.g., bond yield curve).
    - Questionable if quote interpolation is necessary for even the single instrument set, since this results in double interpolation – the first on the quote space, and the second on the span/segment canonical space.
2. <u>Construction from Diverse/Multiple Instrument/Quote Set</u>: Given a diverse set of instruments and/or quotes, we need canonical quote-independent/quote-transforming measure formulation that is valid across the full instrument stretch.
3. <u>Curve Span/Segment Latent State Quantification Metric</u>:

- For discount curves, this can be the discount factor/zero rate/forward rate.
- For forward curves, this can be the absolute forward rate/forward rate basis.
- For credit curves, this can be survival factor/cumulative hazard rate/ forward hazard rate.
- For recovery curves, this can be the expected loss/recovery, of the forward loss/recovery.

4. <u>Cumulative vs. Forward Quantification Metric</u>: The cumulative span quantification metric Z and the forward segment quantification metric Φ are related as $\Phi \Rightarrow \frac{\partial(\mathrm{Z}S)}{\partial S}$, where S is the span variate (specifically the tenor – in this case).
5. <u>Physics of Quantification Metric Constraints</u>: More generally, $\Phi \Rightarrow \Im\left(Z, S, \frac{\partial \mathrm{Z}}{\partial S}\right)$, where $\Im$ comes from the physics of the process. For the discount curve, the credit curve, and the recovery curve $\Im\left(Z, S, \frac{\partial \mathrm{Z}}{\partial S}\right) = \frac{\partial(\mathrm{Z}S)}{\partial S}$.
6. <u>Cumulative Quantification Metric from Forward Quantification Metric (or Span from Segment)</u>: Cumulatives may be extracted from forwards using the quadrature formulation, as they are integrands over the segment dimension. For survival/discount/recovery curves $\mathrm{Z} = \frac{\int_0^t \Phi(S)dS}{t}$.
7. <u>Structure of cumulative vs. Forward</u>: Forward quantification metric is more sharp-edged/swinging than cumulative quantification metric, which, by virtue of the quadrature construct, is smoother.
    - Therefore, single instrument/quote interpolation may be able to use the forward quantification metric, and imply the cumulative quantification metric.
    - Multiple instrument/quote should use the cumulative manifest metric, and perhaps imply the forward quantification metric using the segment <-> span transformation relationship.
8. <u>Constraints on the forward Quantification Metric</u>: Depends on the driver physics.
    - For survival curve, $\Phi \geq 0$, and this is a hard constraint.

- For discount curve, there are no such constraints.
- For recovery curve, the constraint is that $\Phi \geq 0$.

9. <u>Constraints on the cumulative Quantification Metric</u>: Again depends on the stochastic variate driver physics.
    - For survival curve, if Z is the cumulative survival/hazard rate, $Z \geq 0$, and it should be monotonically decreasing - this is a hard constraint.
    - For discount curve, if Z is the discount factor, then $Z \geq 0$. Beyond this there are no constraints.
10. <u>Challenges with interpolating in the forward Quantification Metric space</u>: For survival/discount, due to the exponential nature of the formulation, splining on Φ can very often cause the prior two constraints to be violated – so relatively speaking, the choice is less stable.
11. <u>Span/Segment Quantification Metric Relationship</u>:
    - Discontinuity in the cumulative quantification metric automatically implies discontinuity in the forward quantification metric.
    - Continuous, but non-differentiable cumulative quantification metric implies discontinuity in the forward quantification metric.
    - Continuity in the first derivative of cumulative quantification metric implies continuous, non-differentiable forward quantification metric.
    - Continuity in the first/second/third derivative of cumulative (using, e.g., quartic splines) quantification metric implies continuous, first/second differentiable forward quantification metric.
    - Certain splines become problematic for highly uneven segment lengths, e.g., cubic splines will be unsatisfactory for the situation where you start with close set of nodes and move to a sparser set (Burden and Faires (1997)). This is because the curve is too convex and bulging for points far away from each other.
12. <u>Span Quantification Metric – "Effective" Rate/Hazard Rate</u>: This can simply be defined as $\zeta = -\frac{\log(Z)}{t}$, where Z is either the discount factor (for the discount curve) or the survival factor (for the survival curve). This needs to be matched for 4

powers (quartic) for polynomial spline, or for three derivatives for non-polynomial (e.g., tension) splines.

Calibration Considerations

1. Exponential/Hyperbolic Tension Splines as a Natural Basis for DF representation: This is popular (Sankar (1997), Securities Industry and Financial Markets Association (2004), Andersen (2005)) because the discount factor simply goes as $e^{-\int f dt}$. Obviously this basis will not be suitable for forward/zero rates.
   - The Trouble with the High-Tension Tension Splines is: This causes the segment responses to be almost linear with the predictor, therefore:
     - For big gaps in the predictor ordinates, “linear” can soon become a huge problem.
     - NASTY, NASTY low-tenored forward’s starting near the segment edges.
     - High Tension implies high local forward interest (using above).
     - While Renka (1987) shows an automatic way to extract to specify the tension, the resulting C^1 presents fundamentally no more of an advantage than a C^1 cubic (Le Floch (2013)).
     - Other issues with the impact of automatic selection (see Preuss (1978)) and the corresponding implications for sensitivities remain.
2. Sensitivity of the Forward Rate to the Spot Measure: The forward rate/DF sensitivity to the spot quote is not just low, but also ends up producing multiple matching results.
   - In particular, the presence of root multiplicity within a single segment (as is the case for polynomial splines) reduces the calibration to a needle in a haystack search – with huge demands on intelligent heuristics placed on the searcher.
3. Pay Date DF Pre-computation: This method is outlined in Kinlay/Bai, and is NOT a robust method, for the following reasons:
   - It starts by estimating the DF’s parametrically (using constant forwards) between dates.

- Fine pay date grids (owing to, say, diverse/overlapping instrument types, and diverse/overlapping quote types) means that the interpolation grid becomes highly clustered, and this produces challenges for many splining techniques.

4. Non-linear DV01: The DV01 term $\sum_{j=1}^{n} l_j \Delta_j D_f(t_j)$, or more generally, the DV01-type terms, is non-linear on both the discount factor and the forward rate – this is what makes the curve calibration using the Kinlay/Bai and the Andersen schemes difficult.
   - Relating the discount factor the forward rate as shown may really help simplify the formulation. $D_f(t) = \left\{ \prod_{i=1}^{\eta(t)-1} \left[\frac{1}{1+(t_i - t_{i-1})L_i} \right] \right\} \frac{1}{1+(t - t_{\eta(t)-1})L_{\eta(t)-1}}$. Here $\eta(t)-1$ refers to the instrument maturity that precedes the time t.
5. No Arbitrage Conditions:
   - No Arbitrage for Rates implies that $forwards > 0$ => $\frac{\partial}{\partial t}[r(t)t] \geq 0$, although this can easily seen to be violated in several instances.
   - Options => Arbitrage free Implied Volatility Surface for Call Options (Homescu (2011)) => $\frac{\partial}{\partial t}[C(t,K)] \geq 0$ and $\frac{\partial^2}{\partial K^2}[C(t,K)] \geq 0$.

Curve Construction Formulation

Linearized Discount Curve Calibration from Instruments

1. Cash flow PV Linearity in Discount Factor and Survival: Simply put, $PV = C \times D_f$, or more generally $PV = C \times D_f \times S_P$ where C is the cash flow, D_f is the discount factor, and S_P is the survival probability. The challenge is to re-cast the measure computation in a manner that retains the formulation linearity in the latent state (it is already linear in D_f and S_P, so that simplifies things a bit).
    - Re-casting all the product/measure calibration as a linear equation depends on the product/measure combination, but many typical formulations satisfy this criterion.
2. Different Linearized Discount Curve Formulations:
    - Single Segment Giant Spline => Use all the market observations to construct all the linearization constraints to synthesize one giant multi-basis spline.
    - One Spline Segment per adjacent cash flow pair => This gives maximal control, but ends up being way too computationally involved, as their will be as many spline segments as there are cash flow pairs.
    - One Spline Segment per Instrument Maturity => Here a unique spline segment will be used between 2 adjacent calibration instrument maturities. This ordering is identical to typical instrument level bootstrapping.
    - Transition Spline => This retains the spline cluster per each instrument group. This representation is valuable when you have instruments assembling in cluster (as cash/EDF/swaps etc:, which is obviously a typical arrangement). Judicious choice of knots and instruments etc: reduce the chances of jumps/bumps, although can still be a challenge.
3. Nomenclature:
    - Instrument Set => $l = 1, ..., a$
    - Segment exclusive to instrument l spans the times $\tau_{l-1} \to \tau_l$.

- Instrument l has b cash flows indexed by j: $j \Rightarrow 0,...,b-1$
- Segment l's spline coefficients α_{il} are determined by l's cash flows and market quotes.
- Each Segment has $i = 0,...,n-1$, i.e., n basis function set representing the discount factor.
- Instrument l's cash flow j has a pay date of t_{jl}.

4. Importance of some of the Linear Algebra Operations: While most of what is used in spline systems for linearized curve building can be achieved using a robust linear system solver (e.g., Gauss Elimination, see Press, Teukolsky, Vetterling, and Flannery (1992)), robust matrix inversion algorithms are needed for Jacobian estimation.

Segment Linear Discount Curve Calibration from Instruments

1. Step #1: Identify and sort instruments by their maturities.
   - In between two maturities lies a segment, and the curve start date demarcates the start of the first (exclusive) segment.
2. Step #2: For each instrument, extract the coefficient of each discount factor (which corresponds to the net cash flow at that node).
3. Step #3: Say that the market PV quote of instrument l is Q_l. This indicates

$$Q_l = \sum_{j=0}^{b-1} c_{jl} D_f(t_{jl}) = \sum_{j=0, t_{jl} \le \tau_{l-1}}^{b-1} c_{jl} D_f(t_{jl}) + \sum_{j=0, t_{jl} > \tau_{l-1}}^{b-1} c_{jl} D_f(t_{jl})$$

5. Step #4: Given that all segment l cash flows whose pay date is less than τ_{i-1} belong to the prior periods, their discount factors should be computable. Thus,

   $\mathrm{P}_l = \sum_{j=0, t_{jl} \le \tau_{l-1}}^{b-1} c_{jl} D_f(t_{jl})$ should be pre-computed.

6. <u>Step #5</u>: The segment specific constraint now becomes

$$Q_l = \mathrm{P}_l + \sum_{j=0, t_{jl} > \tau_{l-1}}^{b-1} c_{jl} D_f(t_{jl}) \Rightarrow \sum_{j=0, t_{jl} > \tau_{l-1}}^{b-1} c_{jl} D_f(t_{jl}) = Q_l - \mathrm{P}_l .$$

7. <u>Step #6</u>: In terms of the segment spline coefficients α_{il} and the segment basis functions f_{il}, the constraint gets re-specified as follows:

- $D_f(t_{jl}) = \sum_{i=0}^{n-1} \alpha_{il} f_{il}(t_{jl})$
- $Q_l - \mathrm{P}_l = \sum_{j=0, t_{jl} > \tau_{l-1}}^{b-1} c_{jl} D_f(t_{jl}) \Rightarrow \sum_{j=0, t_{jl} > \tau_{l-1}}^{b-1} c_{jl} \sum_{i=0}^{n-1} \alpha_{il} f_{il}(t_{jl})$
- Again, notice that $\Omega_l = \sum_{j=0, t_{jl} > \tau_{l-1}}^{b-1} \alpha_{jl} f_{jl}(t_{jl})$ can be pre-computed. Thus, the above becomes $\sum_{j=0, t_{jl} > \tau_{l-1}}^{b-1} \alpha_{jl} \Omega_l = Q_l - \mathrm{P}_l$.

8. <u>Step #7</u>: Of course, in general Q_l need not just be the P – it just needs to be any measure linearizable in the discount factor.
9. <u>Cash D_f Coefficient</u>:

- Given a rate calibration measure r_l, $D_f(\tau_l) = \frac{1}{1 + r_l \tau_l}$.

10. <u>EDF D_f Coefficient</u>:

- Given a rate calibration measure r_l, $\frac{-D_f(\tau_{l-1})}{1 + r_l(\tau_l - \tau_{l-1})} + D_f(\tau_l) = 0$.
- Given a price based calibration measure P_l, $-P_l D_f(\tau_{l-1}) + D_f(\tau_l) = 0$.

11. <u>Fixed Stream D_f Coefficient</u>: Given a price measure P_l, $P_l = \sum_{j=0}^{b-1} c\Delta(t_{j-1}, t_j) D_f(t_j)$, where c is the coupon.
12. <u>Floating Stream D_f Coefficient</u>: Given a price measure P_l,

$P_l = \sum_{j=0}^{b-1} s\Delta(t_{j-1}, t_j) D_f(t_j) + [D_f(t_0) - D_f(t_m)]$, where s is the floater spread.

13. IRS D_f Coefficient:

- For a par swap IRS, $Fixed - Floating = 0$ =>

$$\sum_{i=1}^{n} c\Delta(t_{i-1}, t_i) D_f(t_i) - \sum_{j=1}^{m} s\Delta(t_{j-1}, t_j) D_f(t_j) + [D_f(t_0) - D_f(t_m)] = 0 .$$

- Given a price measure P_l,

$$P_l = \sum_{j=0}^{b-1} c\Delta(t_{j-1}, t_j) D_f(t_j) + \sum_{j=0}^{b-1} s\Delta(t_{j-1}, t_j) D_f(t_j) + [D_f(t_0) - D_f(t_m)].$$

14. Bond D_f Coefficient:

- Given a dirty price measure P_l, $P_l = \sum_{j=0}^{b-1} cD_f(t_j) + \sum_{\eta=0}^{N-1} N_j D_f(t_j)$.
- Given a yield measure, the yield can be converted to the dirty price measure P_l.
- Given a spread over TSY measure, it may also be converted to the dirty price measure P_l through the yield.

Curve Jacobian

1. Representation Jacobian: Every Curve implementation needs to generate the Jacobian of the following latent state metric using its corresponding latent state quantification metric:
   - Forward Rate Jacobian to Quote Manifest Measure
   - Discount Factor Jacobian to Quote Manifest Measure
   - Zero Rate Jacobian to Quote Manifest Measure
2. Importance of the representation Self-Jacobian: Representation Self-Jacobian computation efficiency is critical, since Jacobian of any function $F(Y)$ is going to be dependent on the self-Jacobian $\frac{\partial Y(t)}{\partial Y(t_K)}$ because of the chain rule.
3. Forward Rate->DF Jacobian:

- $F(t_A, t_B) = \frac{1}{t_B - t_A} \ln\left(\frac{\partial D_f(t_A)}{\partial D_f(t_B)}\right)$.
- $\frac{\partial F(t_A, t_B)}{\partial D_f(t_k)} = \frac{1}{t_B - t_A}\left\{\frac{1}{D_f(t_A)}\frac{\partial D_f(t_A)}{\partial D_f(t_k)} - \frac{1}{D_f(t_B)}\frac{\partial D_f(t_B)}{\partial D_f(t_k)}\right\}$.
- $F(t_A, t_B)$ => Forward rate between times t_A and t_B.
- $D_f(t_k)$ => Discount Factor at time t_k

4. <u>Zero Rate to Forward Rate Equivalence</u>: This equivalence may be used to construct the Zero Rate Jacobian From the Forward Rate Jacobian. Thus the above equation may be used to extract the Zero Rate micro-Jacobian.
5. <u>Zero Rate->DF Jacobian</u>:
   - $\frac{\partial Z(t)}{\partial D_f(t_k)} = \frac{1}{t - t_0}\left\{\frac{1}{D_f(t)}\frac{\partial D_f(t)}{\partial D_f(t_k)}\right\}$
   - $Z(t)$ => Zero rate at time t
6. <u>Analytical Sensitivity vs. Quote Bumped Sensitivity</u>: In general, when dealing with the splined mechanisms for curve cooking, it may not be accurate to depend on the quote bumped sensitivity, because it may end up throwing it to a totally different curve builder scheme (Le Floc'h (2013)).
   - Also, analytical sensitivities may be estimated right during the calibration itself. However, analytical-to-quote sensitivities implies two-stage Jacobian – the Jacobian of the quote to the state representations, then the Jacobian of the state representation to the sensitivity measure.
   - In-situ Calibration Sensitivites => Measure to state sensitivities maybe generated quiet readily, depending on the calibration mode.
     - For linear calibrator, this is simply the state Jacobian inverse.
     - In some non-linear search techniques (esp. open ones like the Newton's method, but with the closed schemes as well), sensitivity Jacobians are automatically (or using light adjustment) generated as part of the calibration itself.
   - Spline coefficient sensitivity to segment/node inputs => High sensitivity of the spline coefficients to the node inputs across specific stretches indicates

instability in curve (re-) construction and the corresponding deltas (i.e., spurious deltas and leakage). Le Floc'h (2013) examines this for several standard interpolating estimators in use.

7. <u>Derivative to Quote Jacobian via the Discount Factor Latent State</u>:
   - $c \Rightarrow 0,...,d-1$ Calibration Components
   - $q_c \Rightarrow q_0,...,q_{d-1}$ Corresponding Quotes
   - Let's say the Derivative PV is $P = \sum_{j=1}^{m} \Diamond_j D_f(t_j) \Rightarrow \frac{\partial P}{\partial q_c} = \sum_{j=1}^{m} \Diamond_j \frac{\partial D_f(t_j)}{\partial q_c}$. Thus what is typically needed to estimate product-to-quote sensitivities via the Discount Factor latent state is $\frac{\partial D_f(t_j)}{\partial q_c}$.
8. <u>Quote->Zero Rate Jacobian</u>:
   - $\frac{\partial Q_j(t)}{\partial Z(t_k)} = (t_k - t_0)\left\{ D_f(t_k) \frac{\partial Q_j(t)}{\partial D_f(t_k)} \right\}$
   - $Z(t)$ => Zero rate at time t
9. <u>PV->Quote Jacobian</u>:
   - $\frac{\partial PV_j(t)}{\partial Q_k} = \sum_{i=1}^{n} \left\{ \frac{\partial PV_j(t)}{\partial D_f(t_i)} \div \frac{\partial Q_j(t)}{\partial D_f(t_i)} \right\}$
10. <u>Cash Rate DF micro-Jacobian</u>:
    - $\frac{\partial r_j}{\partial D_f(t_k)} = -\frac{1}{\partial D_f(t_j)} \frac{1}{t_j - t_{START}} \frac{\partial D_f(t_j)}{\partial D_f(t_k)}$
    - r_j => Cash Rate Quote for the jth Cash instrument.
    - $D_f(t_j)$ => Discount Factor at time t_j
11. <u>Cash Instrument PV-DF micro-Jacobian</u>:
    - $\frac{\partial PV_{CASH,j}}{\partial D_f(t_k)} = -\frac{1}{\partial D_f(t_{j,SETTLE})} \frac{\partial D_f(t_j)}{\partial D_f(t_k)}$
    - There is practically no performance impact on construction of the PV-DF micro-Jacobian in the adjoint mode as opposed to the forward mode, due to the triviality of the adjoint.

12. Euro-dollar Future DF micro-Jacobian:

- $$\frac{\partial Q_j}{\partial D_f(t_k)} = \frac{\partial D_f(t_j)}{\partial D_f(t_k)} \frac{1}{\partial D_f(t_{j,START})} - \frac{D_f(t_j)}{D_f^{\,2}(t_{j,START})} \frac{\partial D_f(t_{j,START})}{\partial D_f(t_k)}$$
- Q_j => Quote for the jth EDF with start date of $t_{j,START}$ and maturity of t_j.

13. Euro-dollar Future PV-DF micro-Jacobian:

- $$\frac{\partial PV_{EDF,j}}{\partial D_f(t_k)} = \frac{\partial D_f(t_j)}{\partial D_f(t_k)} \frac{1}{\partial D_f(t_{j,START})} - \frac{D_f(t_j)}{D_f^{\,2}(t_{j,START})} \frac{\partial D_f(t_{j,START})}{\partial D_f(t_k)}$$
- There is practically no performance impact on construction of the PV-DF micro-Jacobian in then adjoint mode as opposed for forward mode, due to the triviality of the adjoint.

14. Interest Rate Swap DF micro-Jacobian:

- $Q_j DV01_j = PV_{Floating,j}$
- Q_j => Quote for the jth IRS maturing at t_j.
- $DV01_j$ => DV01 of the swap
- $PV_{Floating,j}$ => Floating PV of the swap
- $$\frac{\partial \lfloor Q_j DV01_j \rfloor}{\partial D_f(t_k)} = \frac{\partial \lfloor PV_{Floating,j} \rfloor}{\partial D_f(t_k)}$$
- $$\frac{\partial \lfloor Q_j DV01_j \rfloor}{\partial D_f(t_k)} = \frac{\partial Q_j}{\partial D_f(t_k)} DV01_j + Q_j \frac{dDV01_j}{\partial D_f(t_k)}$$
- $$\frac{dDV01_j}{\partial D_f(t_k)} = \sum_{i=1}^{j} N(t_i)\Delta_i \frac{\partial D_f(t_i)}{\partial D_f(t_k)}$$
- $$PV_{Floating,j} = \sum_{i=1}^{j} l_i N(t_i)\Delta_i D_f(t_i)$$
- $$\frac{\partial PV_{Floating,j}}{\partial D_f(t_k)} = \sum_{i=1}^{j} N(t_i)\Delta_i D_f(t_i) \frac{\partial l_i}{\partial D_f(t_k)} + \sum_{i=1}^{j} l_i N(t_i)\Delta_i \frac{\partial D_f(t_i)}{\partial D_f(t_k)}$$

15. Interest Rate Swap PV-DF micro-Jacobian: See Hull (2002) for the preliminaries.

- $$\frac{\partial PV_{IRS,j}}{\partial D_f(t_k)} = \sum_{i=1}^{j} N(t_i)\Delta(t_{i-1},t_i)\left\{(c_j - l_i)\frac{\partial D_f(t_i)}{\partial D_f(t_k)} - D_f(t_i)\frac{\partial l_i}{\partial D_f(t_k)}\right\}$$

- There is no performance impact on construction of the PV-DF micro-Jacobian in then adjoint mode as opposed for forward mode, due to the triviality of the adjoint. Either way the performance is $\Theta(n \times k)$, where n is the number of cash flows, and k is the number of curve factors.

16. <u>Credit Default Swap DF micro-Jacobian</u>:
    - $PV_{CDS,j} = PV_{Coupon,j} - PV_{LOSS,j} + PV_{ACCRUED,j}$
    - j => jth CDS Contract with a maturity t_j
    - c_j => Coupon of the jth CDS
    - $PV_{CDS,j}$ => PV of the full CDS contract
    - $PV_{Coupon,j}$ => PV of the Coupon leg of the CDS Contract
    - $PV_{ACCRUED,j}$ => PV of the Accrual paid on default
    - $PV_{Coupon,j} = c_j \sum_{i=1}^{j} N(t_i)\Delta_i S_P(t_i) D_f(t_i)$
    - $\frac{\partial PV_{Coupon,j}}{\partial D_f(t_k)} = c_j \sum_{i=1}^{j} N(t_i)\Delta_i S_P(t_i) \frac{\partial D_f(t_i)}{\partial D_f(t_k)} + \frac{\partial c_j}{\partial D_f(t_k)} \sum_{i=1}^{j} N(t_i)\Delta_i S_P(t_i) D_f(t_i)$
    - $PV_{LOSS,j} = \int_0^{t_j} N(t)[1 - R(t)] D_f(t) dS_P(t)$
    - $\frac{\partial PV_{LOSS,j}}{\partial D_f(t_k)} = \int_0^{t_j} N(t)[1 - R(t)] \frac{\partial D_f(t)}{\partial D_f(t_k)} dS_P(t)$
    - $PV_{ACCRUED,j} = c_j \sum_{i=1}^{j} \int_{t_{i-1}}^{t_i} N(t)\Delta(t, t_{i-1}) D_f(t) dS_P(t)$
    - $\frac{\partial PV_{ACCRUED,j}}{\partial D_f(t_k)} = \frac{\partial c_j}{\partial D_f(t_k)} \sum_{i=1}^{j} \int_{t_{i-1}}^{t_i} N(t)\Delta(t, t_{i-1}) D_f(t) dS_P(t) + c_j \sum_{i=1}^{j} \int_{t_{i-1}}^{t_i} N(t)\Delta(t, t_{i-1}) \frac{\partial D_f(t)}{\partial D_f(t_k)} dS_P(t)$

17. <u>Credit Default Swap DF micro-Jacobian</u>:

- $$\frac{\partial PV_{CDS,j}}{\partial D_f(t_K)} = c_j \sum_{i=1}^{j} \left\{ \left[N(t_i)\Delta(t_{i-1},t_i)S(t_i)\frac{\partial D_f(t_i)}{\partial D_f(t_k)} \right] + \int_{t_{i-1}}^{t_i} N(t)\left[c_j\Delta(t_{i-1},t) - \{1 - R(t)\}\right] dP(t) \right\}$$

- There is no performance impact on construction of the PV-DF micro-Jacobian in then adjoint mode as opposed for forward mode, due to the triviality of the adjoint. Either way the performance is $\Theta(n \times k)$, where n is the number of cash flows, and k is the number of curve factors.

Spanning Spline

Formulation and Set up

1. Spline vs. Boot Span: For the purposes of this discussion, the main difference between spline and boot span is that, in boot span, the segment boundaries HAVE to line up with the instrument maturity edges. In spline spans, however, additional criterion-based knots may be used to determine the boundaries (e.g., parametric knot insertion in line with regression spline approaches).
2. Basic Setup: All instruments and quotes fall into one set of constraints as $\sum_{j=0}^{b-1} c_{jl} D_f(t_{jl}) = Q_l$, where $l = 1,...,a$.
   - In general, $a < b$, so you have $b - a$ degrees of freedom.
3. Local Ordinate Re-formulation: The spline extends from $t_{START} \rightarrow t_{b-1}$. Setting $x_i = \frac{t_i - t_{START}}{t_{b-1} - t_{START}}$, $\sum_{j=0}^{b-1} c_{jl} D_f(t_{jl}) \Rightarrow \sum_{j=0}^{b-1} c_{jl} D_f(x_{jl})$. Further, $D_f(t_{START}) = D_f(x = 0) = 1$.
4. Basis Formulation: Setting $D_f(x) = \sum_{i=0}^{n-1} \alpha_i f_i(x)$, $\Rightarrow \sum_{j=0}^{b-1} c_{jl} \sum_{i=0}^{n-1} \alpha_i f_i(t_j) = Q_l \Rightarrow \sum_{i=0}^{n-1} \alpha_i \left[\sum_{j=0}^{b-1} c_{jl} f_i(t_j) \right] = Q_l$. Thus, if $n = a$, there now are a equations and a unknowns.
5. Monotonicity Preservation in Spanning Splines: The heterogeneity of the calibration instruments demands special techniques for monotonicity maintenance (Hagan West (2006) described in detail earlier was a sample).
   - Stringent monotonic constraints introduced by Hyman (1983) was relaxed by Dougherty, Edelman, and Hyman (1989), and this was works well in practice in its ability to maintain monotonicity (Ametrano and Bianchetti (2009), Le Floc'h (2013), also implemented in Quantlib (2009)).

- Intermediate filter constraints introduced by Steffen (1990) and their variants treated in some detail by Huynh (1993) – all suffer from the same unnatural "dip"s or cook bumps.

6. Pros: As always, the degrees of freedom may be expanded beyond a to allow for optimizing spline construction (covered in the spline builder section).
7. Cons: With many basis functions (esp. for polynomials), the inevitable Runge's phenomenon takes over.

Challenges with the Spanning Spline Approach

1. Problems with Cubic Polynomial Spline: Too well known to documented – spurious inflection, too much concavity/convexity at widely separated predictor nodes (esp. in long end), and no guarantee of positivity where desired.
    - As noted in Le Floc'h (2013), monotone variants (including Hagan and West (2006), Wolberg and Alfy (1999), Hyman (1983)) of the standard cubic spline have differing degrees of problems since they are attempt to model the entire span with a single representation.
2. Problems with Quartic Spline: While this makes the interpolation very smooth (Adams and van Deventer (1994), van Deventer and Inai (1997), Adams (2001), Lim and Xiao (2002), Quant Financial Research (2003)), the stiffness needed for shape-preservation is completely lost. Other troubles as with cubic splines (spurious inflection, too much concavity/convexity at widely separated predictor nodes (esp. in long end), and no guarantee of positivity where desired) as well Runge's swings are also present.

Monotone Decreasing Splines

Motivation

1. These are spline basis functions that monotonically decrease over the given interval. Valuable for representing discount factors.
2. Why represent discount factors? Because the pay-offs are linearizable in them, so working with them implies working with the linear rates space representation, and all the advantages that come with that.

Exponential Rational Basis Spline

1. Basis Function Set: $\left\{1, \frac{1}{1+t}, e^{-t}, \frac{e^{-t}}{1+t}\right\}$
2. Monotone Decreasing Nature: Each of the above basis functions is decreasing. For the functional form to be monotonically decreasing, conservatively speaking, this imposes the demand that $\{\beta_i = 0\}$ for every i.
   - Alternatively, we may also require that no infection exist within the given segment, but that is hard to enforce.

Exponential Mixture Basis Set

1. Motivation: Since the discounting function goes as e^{-t}, an exponential mixture basis such as $e^{-\lambda_i t}$ may be a good choice, as they are both intuitively monotone, and linear combinations of them produce convexity/concavity.
2. Basis Function Set: $\left\{e^{-\lambda_i t}\right\}$ for $i \in 0, ..., n-1$.

- Choosing λ_i: Since for C^2 continuity we require 4 basis functions, we choose $\lambda_{Floor} = 0$, λ_{Low}, λ_{Medium}, and λ_{High}. $\lambda_{Floor} = 0$ accounts for adjusting jumps.
- Typical values can be: $\lambda_{Floor} = 0$, $\lambda_{Low} = 1\%$, $\lambda_{Medium} = 5\%$, and $\lambda_{High} = 25\%$.
- Parallel with Tension Splines => λ_i are comparable to tension splines.
- With this choice, C^k may be maintained for $k \geq 2$, thereby making the forwards continuous, preserving locality, imparting segment convexity/concavity. Thus all the smoothing schemes may be maintained.

3. Similarity with exponential/hyperbolic tension splines: Very similar in formulation. However, given that with exponential/hyperbolic basis set spline at one of basis functions has a non-negative exponential argument, that basis function becomes monotonically increasing.
   - Further, while estimation of the exponential tension needs to be done extraneously (Renka (1987)), here we appeal to the intuitive physics, as shown.

Hagan West (2006) Smoothness Preserving Spanning Spline

Monotone/Convexity Preserving Estimator

1. Premise: This is primarily focused on a quadratic interpolant, but it also contains heterogeneously inserted sub-segment knots in effect to achieve the desired monotonicity, convexity, and positivity effect.
2. Philosophy:
   - This is mainly meant for forward rates inside finance, although bit more general outside of it.
   - The observation set $\{z_i\}_{i=1}^{n}$ is simply a quantity conserved on a per-segment basis, e.g., the segment mean of the state variate response, i.e., $z_i = \frac{1}{\tau_i - \tau_{i-1}} \int_{\tau_{i-1}}^{\tau_i} y(t)dt$.
   - $y(t)$ is positive and piece-wise quadratic inside of $[\tau_{i-1}, \tau_i]$.
   - The node response value y_i at the predicate ordinate τ_i is linearly interpolated from the observations at z_i and z_{i+1} (obviously edges will be treated slightly differently).
   - Based on the specified monotonicity maintenance and convexity preservation criteria, the algorithm identifies and inserts knots. Zero or more knots may need to be inserted.
   - The quadratic interpolant is essentially a Bessel C^1 Hermite interpolant.
   - Finally, similarity response value may be applied for positivity, and range-bounded-ness.
3. Steps:
   - Infer the response node value y_i at the predicate ordinate τ_i is linearly interpolated from the observations at z_i and z_{i+1} as:
     - $y_i = \frac{\tau_i - \tau_{i-1}}{\tau_{i+1} - \tau_{i-1}} z_{i+1} + \frac{\tau_{i+1} - \tau_i}{\tau_{i+1} - \tau_{i-1}} z_i$ for $i \neq 0, n$

- $y_0 = z_1 - \frac{1}{2}[y_1 - z_1]$
- $y_n = z_n - \frac{1}{2}[y_{n-1} - z_n]$

- Work out the "Z-score" metric within $[\tau_{i-1}, \tau_i]$:
  - $g_{i-1} = y(\tau_{i-1}) - z_i = y_{i-1} - z_i$
  - $g_i = y(\tau_i) - z_i = y_i - z_i$
  - Further, we work in the local predictor ordinate space x, where $x = \frac{\tau - \tau_{i-1}}{\tau_i - \tau_{i-1}}$.
- Apply the appropriate adjustments for the monotonicity/convexity enforcement at the appropriate zones:
  - Case $g_{i-1} > 0$, $-\frac{1}{2}g_{i-1} \ge g_i \ge -2g_{i-1}$ [OR] $g_{i-1} < 0$, $-\frac{1}{2}g_{i-1} \le g_i \le -2g_{i-1}$: Here, the function $g(\tau) = g_{i-1}(1 - 4x + 3x^2) + g_i(-2x + 3x^2)$ can be used unchanged, as the original construct is already monotone and convex.
  - Case $g_{i-1} > 0$, $g_i \ge -2g_{i-1}$ [OR] $g_{i-1} < 0$, $g_i \le -2g_{i-1}$: Here, insert a knot at $\eta = \frac{g_i + 2g_{i-1}}{g_i - g_{i-1}}$. The segment univariate now becomes: $g(\tau) = g_{i-1}$ for $0 \le x \le \eta$, and $g(\tau) = g_{i-1} + (g_i - g_{i-1})\left[\frac{x - \eta}{1 - \eta}\right]^2$ for $\eta < x \le 1$.
  - Case $g_{i-1} > 0$, $0 > g_i > -\frac{1}{2}g_{i-1}$ [OR] $g_{i-1} < 0$, $0 < g_i < -\frac{1}{2}g_{i-1}$: Here, insert a knot at $\eta = \frac{3g_{i-1}}{g_i - g_{i-1}}$. The segment univariate now becomes: $g(\tau) = g_{i-1} + (g_i - g_{i-1})\left[\frac{\eta - x}{\eta}\right]^2$ for $0 \le x < \eta$, and $g(\tau) = g_i$ for $\eta \le x \le 1$.

- Case $g_{i-1} \ge 0$, $g_i \ge 0$ [OR] $g_{i-1} \le 0$, $g_i \le 0$: Here, insert a knot at $\eta = \frac{g_i}{g_i + g_{i-1}}$. Setting $\mathrm{A} = -\frac{g_{i-1}g_i}{g_i + g_{i-1}}$, the segment univariate now becomes: $g(\tau) = \mathrm{A} + (g_{i-1} - \mathrm{A})\left[\frac{\eta - x}{\eta}\right]^2$ for $0 \le x < \eta$, and $g(\tau) = \mathrm{A} + (g_i - \mathrm{A})\left[\frac{x - \eta}{1 - \eta}\right]^2$ for $\eta \le x \le 1$.

Positivity Preserving Estimator

1. <u>Positivity of the interpolant</u>: Hagan and West (2006) guarantee this by setting he following bounds:
   - $y_0 = bound[0, y_0, 2z_1]$
   - $y_n = bound[0, y_n, 2z_n]$
   - $y_i = bound[0, y_i, 2 * \min(z_i, z_{i+1})]$ for $i \ne 0, n$

Ameliorating Estimator

1. <u>Amelioration (i.e., Smoothing) of the Interpolant - Steps</u>:
   - #1: Expand the Range at the edges => Add an interval at the beginning and at the end.
     - $\tau_{-1} = \tau_0 - (\tau_1 - \tau_0)$ and $z_0 = z_1 - \frac{\tau_1 - \tau_0}{\tau_2 - \tau_0}(z_2 - z_1)$
     - $\tau_{n+1} = \tau_n + (\tau_n - \tau_{n-1})$ and $z_{n+1} = z_n + \frac{\tau_n - \tau_{n-1}}{\tau_n - \tau_{n-2}}(z_n - z_{n-1})$
     - Complete the linear interpolation of the response variate across all the intervals as before.

- #2: Set the Extraneous Bounds Parametrically/Empirically => Assume that the left and the right mini-max bounds are set extraneously for each segment, i.e., $y_{i,LeftMin}$, $y_{i,LeftMax}$, $y_{i,RightMin}$, and $y_{i,RightMax}$ are extraneously set. They may be set either point-by-point, or using another parametrization. This ensures locality, at expense of C^k, however.
  - Check if the given response value is inside of the specified range, i.e., $\min(y_{i,LeftMax}, y_{i,RightMax}) \geq y_i \geq \max(y_{i,LeftMin}, y_{i,RightMin})$, set as follows:
    - If $y_i < \min(y_{i,LeftMax}, y_{i,RightMax})$, $y_i = \min(y_{i,LeftMax}, y_{i,RightMax})$.
    - If $y_i > \max(y_{i,LeftMin}, y_{i,RightMin})$, $y_i = \max(y_{i,LeftMin}, y_{i,RightMin})$.
  - Otherwise:
    - If $y_i < \min(y_{i,LeftMax}, y_{i,RightMax})$, $y_i = \min(y_{i,LeftMax}, y_{i,RightMax})$.
    - If $y_i > \max(y_{i,LeftMin}, y_{i,RightMin})$, $y_i = \max(y_{i,LeftMin}, y_{i,RightMin})$.
- #3: Re-work the edges =>
  - If $|y_0 - z_0| > \frac{1}{2}|y_1 - z_0|$, then $y_0 = z_1 - \frac{1}{2}|y_1 - z_0|$.
  - If $|y_n - z_n| > \frac{1}{2}|y_{n-1} - z_n|$, then $y_n = z_n + \frac{1}{2}|y_{n-1} - z_n|$.
  - If y_0 is already explicitly specified (as the zero-day rate in some markets) use that instead.
  - Finally, if needed re-apply the positivity enforcement across all the segments as before.

Harmonic Spline Extension to the Framework above

1. Harmonic Splines and Continuous Limiters extension: Le Floc'h (2013) applies the harmonic splines originally introduced by Fritsch and Butland (1984), and extends the monotonicity preserving limiters of Van Leer (1974) and Huynh (1993) by using rational functions.

2. <u>Harmonic Forwards in Hagan-West</u>: Couple of interesting items to note: Given $m_{i+1} = \frac{y_{i+1} - y_i}{t_{i+1} - t_i}$, on substituting $y_i = -z_i t_i$, you get $z_{i+1} = -m_i$, and $-s_i = -+f_i$.
3. <u>Estimation of the node forwards using Harmonic mean</u>: Apply the above now to get $\frac{1}{f_i} = \frac{t_i - t_{i-1} + 2(t_{i+1} - t_i)}{3(t_{i+1} - t_{i-1})} \frac{1}{z_i} + \frac{t_{i+1} - t_i + 2(t_i - t_{i-1})}{3(t_{i+1} - t_{i-1})} \frac{1}{z_{i+1}}$ if $z_i z_{i+1} > 0$, and $f_i = 0$ otherwise. After this, the regular Hagan-West may be applied without the need to enforce monotonic or convexity constraints, as it now is monotonic/convex by construction.

Minimal Quadratic Estimator

1. <u>Design Philosophy</u>: The algorithm extracts the spline coefficients keeping in mind the following:
   - Formulate using a 2nd degree quadratic polynomial for each segment
   - Maintain the Conserved Quantities
   - Maintain the Segment Edge Continuities
   - Optimize for the linear combination of two penalties:
     - Jump of the inter-segment discontinuities on the first derivatives
     - Curvature of the second derivative
2. <u>Step #1: Preservation of the Conserved Quantity Set</u>: This results in the following equation: $z_i = a_i + \frac{1}{2} b_i h_i + \frac{1}{3} c_i h_i^2$
3. <u>Step #2: Edge Continuity Constraint</u>: $a_{i+1} = a_i + b_i h_i + c_i h_i^2$.
4. <u>Step #3: Minimize the Penalty</u>:
   - Jump of the inter-segment discontinuities on the first derivatives
     $$J_{1i} = [b_i + 2c_i h_i - b_{i+1}]^2 = [(b_i - b_{i+1}) + 2c_i h_i]^2 = (b_i - b_{i+1})^2 + 4c_i h_i (b_i - b_{i+1}) + 4c_i^2 h_i^2$$
   - Curvature of the second derivative $J_{2i} = [h_i \bullet 2c_i]^2 = 4c_i^2 h_i^2$

- Complete Penalty Formulation =>

  $P(\omega) = \omega J_{2i} + (1-\omega) J_{2i} = [h_i \bullet 2c_i]^2 = 4c_i^2 h_i^2 + 4\omega c_i h_i (b_i - b_{i+1})$
- $\frac{\partial P(\omega)}{\partial c_i} = 0 \Rightarrow 8c_i h_i^2 + 4\omega c_i h_i (b_i - b_{i+1}) = 0 \Rightarrow \frac{\partial P(\omega)}{\partial c_i} = 8h_i^2 > 0$, so minimum exists.

5. Equation Set and Unknowns Analysis:

- $z_i = a_i + \frac{1}{2} b_i h_i + \frac{1}{3} c_i h_i^2$ => One per segment => $(n-1)$ Equations
- $a_{i+1} = a_i + b_i h_i + c_i h_i^2$ => One per common edge => $(n-2)$ Equations
- $2c_i h_i^2 + \omega c_i h_i (b_i - b_{i+1}) = 0$ => One each for all c_i up to c_{n-2} => $(n-2)$ Equations
- Total number of linear equations => $3n-5$
- Total number of unknowns => $3n-3$
- As always, the final 2 conditions from natural, financial, or the not-a-knot clamped boundary conditions.

Penalizing Closeness of Fit and Curvature Penalty

1. Basic Setup: As described in the companion Spline Library Documentation,
   - Gross Penalizer = Fitness Match Penalizer + Curvature Penalizer
   - $\Re = \frac{1}{q}\Re_F + \lambda\Re_C$
   - $\Re_F = \sum_{p=0}^{q-1} W_p (y_p - Y_p)^2$
   - $\Re_C = \int_{x_l}^{x_{l+1}} \left(\frac{\partial^m y}{\partial x^m}\right)^2 dx$
2. Estimation of λ: While the segment spline coefficients are computed by minimizing $\Re$, λ is often extraneously supplied as a tuner that trades the prefect high degree of fit to the curvature. Tanggaard (1997) suggests using a few methods to estimate λ:
   - Using the GCV criterion as demonstrated by Craven and Wahba (1979) and Wahba (1990).
   - From the smoothing spline viewpoint, set the number of basis functions, then search for the corresponding λ using the technique listed in Tanggaard (1997).
3. Measurement Filtering vs. Best Fit Weighted Response: These approaches are very similar, in that the Best Fit Weighted Response "steers" the calibrated spline basis and their coefficients to accommodate the measurements in the uncertain sense (potentially by incorporating measurement uncertainty).
   a. If the measurement uncertainty/variance is explicitly known, the Andersen (2005), the Tanggaard (1997), and/or the GCV techniques may be used to extract better estimate for λ - through Andersen RMS γ^2 estimator, Craven/Wahba's GCV, or Tanggaard's trace-based λ estimator.
   b. Differences => However, it needs to be remembered that, for current curve construction methodologies, a key requirement is the γ^2 matches (i.e., exactly

reproducing state estimations) – which is not the typical case for the filtered state estimations.

4. Effectiveness of State Representation Quantification Metric: The combination of curvature penalty, the length penalty, and the closeness of fit penalty must be taken together to gauge the effectiveness of the chosen Quantification Metric/Smoothing spline scheme set. Alternatively, full simulations of the manifest metric (with induced noise terms as explained in for e.g., Fisher, Nychka, and Zervos (1994)) and their corresponding evaluations are also appropriate, although they tend to be time consuming (and possibly overkill).

Extrapolation in Curve Construction

1. Latent State Choice for the Extrapolator: The quantification metric used to extrapolate the latent state may be completely different from that used to infer within the span.
   - This clearly indicates that the span spans the extrapolated range as well. Further, the extrapolator should be a property of the Span, not any stretch.
2. Extrapolator Construction: At the span edges, the C^k continuity constraints may be passed onto the extrapolator as well. These may take the form of the stretch boundary conditions (natural/financial etc).
3. State Space Extrapolation using Synthetic Observations: This is really what it is. In particular, to get the desired left/right boundary behavior, you may insert synthetic observations at either end to produce the desired custom behavior (this may also be used in lieu of the explicit boundary condition specification).

Multi-Pass Curve Construction

Motivation

1. Introduction: This is composed of one shape preserving pass on the inferable state quantification metric, followed by on or more "smoothing passes".
2. Shape Preserving Pass: The shape preservation pass occurs on the "native designate" measure, preferably one that is linearly inferred from the manifest measure. The primary objective of the shape preservation pass is to maintain the monotonicity, the convexity, the locality, and possibly the positivity of the quantification metric.
   - The output of the shape-preserving pass is a span on the quantification metric that is "well-behaved", and one that contains a new set of "truthness" nodes on which the eventual smoothing can be done.
3. Shape Preservation Variants:
   - Linear in the discount Factor Quantification Metric => They are obviously the best shape preserver (owing to the perfection in the match and zero curvature penalty), but they no inherent convexity/concavity in them, so it gets harder fort the smoothing stage.
   - Constant forward rate bootstrapping may also be used.
4. Smoothing Pass: Here you smooth on the appropriate quantification metric that is deemed to be a better hidden-state characterizer.
5. Advantages of the Shape-Preserving Pass:
   - Separation between Shape-preservation and smoothing.
   - Choice of convenient, yet potentially different metrics across shape-preserving and smoothing.
   - The final state representation quantification metric need not be linear on the manifest measure.

- The granularity/precision of fit of the curve automatically adjusts with information (i.e., cash flow event dates such as pay dates), thereby making it inherently more precise.
- PCHIP techniques may be applied more conveniently on the smoothing pass.
- Other closeness of fit techniques (such as least squares methodologies, etc:) become much more relevant on the smoothing pass.

6. Disadvantages of the Shape-Preserving Pass:
   - Calculation overhead penalty associated with the dual pass (although, by choosing linearity between manifest measure/quantification metric and the quantification metric/ quantification metric combinations this adverse impact maybe reduced).
   - Artifacts produced during shape-preservation (again, there will be artifacts associated with just about any basis representation).

Bear Sterns Multi-Pass Curve Building Techniques

1. DENSE Methodology: This method is outlined in Nahum (2004).
   - Cash/Forwards => Piece-wise constant forwards. Turn Spreads imposed as needed.
   - Swaps => Shape Preserving uniform tension splines.
   - RAW Swaps Inputs => Quarterly swap rates are now re-implied from the curve constructed in the earlier stage.
   - From these new swap quotes, a new curve is constructed using quarterly constant forward rates (constant forward rates methodology is called RAW).
2. DUAL DENSE Methodology: Again, this method is outlined in Nahum (2004).
   - Short end (Cash/Futures) => Daily forwards (i.e., constant daily forwards or cdf) latent state implied.
   - Long End => Same methodology as DENSE, except for the non-uniform tension that is applied across quarterly swap contracts.

Transition Spline (Or Stitching Spline)

Motivation

1. Spline per Instrument Grouping: Another possibility is to use transition spline to bridge across different instrument groups – this simply needs to adjust to the smoothness/truthness constraints of each of the instrument groups.
   - Essentially, transition splines connect spline families across instrument group (each instrument essentially belongs to its own spline cluster).
2. Design:
   - May use discontinuous Hermite splines in the transition area, or higher order basis (say, with an appropriate C^k constraint), or even an optimizing transition spline.
   - Instrument choice is critical if we are to avoid steep transition slopes (esp. tight group gaps, and steep measure drops). These are challenges in any mechanism, but possibly a lot more here.
   - Construct single instrument spanning spline curves, then demarcate/spec out the instrument range, finally bridge in the transition splines.
   - Transition splines may also be used to stitch in arbitrary instruments together, each belonging to its own separate group, although it is hard to find a practical need for such a construct.
   - In general, instrument group boundaries need not strictly coincide with the instrument termination nodes (esp. in case of stitch-in splines). Boundaries may be inserted using any of the appropriate knot insertion techniques.
3. Advantages:
   - These preserve the curve character embedded in each instrument grouping, which can be a sub-set of a vaster instrument set.
   - By retaining the localization to the corresponding instrument grouping, the hedges produces by the transition spline may, in principle, be better than those produced by the typical ones.

4. Disadvantages:
    - Of course, by construction, they do not allow for overlapping instrument groups (which, however, may not be a problem in the practical world). This forces a decision on the instrument set choices and boundaries.
    - Technically, the single "natural spline boundary condition" is not applicable across all the unprocessed instrument groups – this is really what is compromised.
        - How much the effectiveness is compromised due to the above may be estimated using targeted metrics, say the span DPE.
5. Transition Segment in the Transition Spline: This needs at least $2k+2$ basis functions for representation, as it needs to "mate out" the left stretch and the right stretch ($k+k$ for each of the C^k continuity spec - plus 2 more, one at each end to match up the point node).
6. Using Transition Splines for Calibration Instrument Selection: As shown in Figures 2 and 3 below, the transition stretch represented in figure 2 is narrower, and therefore more abrupt/jumpy (with corresponding implications for the forward rates) than that in Figure 3. A criteria based approach is necessary to develop this.

Stretch Modeling Using Transition Splines

1. Information Propagation across Stretches: All the truthness/smoothness information of the predecessor stretch is captured by the stretch's calibrated span parameters. Any state inference for predictors in a given domain needs to be deferred to the domain's span stretch.
    - The corollary to the above is that trailing stretches will typically need information from the leading stretches for state inference/estimation (leading/trailing here are set in regards to the inference flow (or information flow)). Applied to discount curve cooking, the leading stretch that uses cash instruments is essentially self-calibrating, whereas the trailing stretch of swap instruments is going to rely on information that comes out of the cash calibration. Going into swap segments, the

information will propagated in the form of RVC's, so they will need to be handled right from the left-most segment of each stretch.

- Regular Stretches vs. Finance Curve Stretches => For typical stretch construction, all you need is the transmission of the segment-to-segment continuity constraints through C^k. For segment curve builders, however, $Constraint(Segment_i) = f(Segment_0, ..., Segment_{N-1})$, i.e., more construction information in addition to just the C^k is required (mostly via explicit evaluation of arbitrary points in earlier segments' stretches).

2. Response Stretches: Markov response state variables may follow distinct behavior in different predictor stretches. For example, the discount factor/zero rate/swap rate may be characterized using one set of representations for the cash stretch, whereas the swap stretch may use a different set.
3. Why Response Stretches exist: Is it simply because of the instrument choice (cash for the front end, swap for the back end, etc:), or is there a more fundamental driver? Can't say one way or the other, but the fact is we empirically attempt to match point-by-point in a left to right manner (we do this today) without compromising the empirical characteristics of each instrument group. We call each of these groups manifest groups, since they could be result of specific product manifest measures).
4. Manifest Group Contribution to the Response Signal Strength: Say that a signal strength contribution to a specific response signal is proportional to its liquidity (to improve accuracy, you may make it sided liquidity). As you move from left to right in the predictor space, by working it in terms of the liquidity-fade of the left stretch to the liquidity-explode of the right stretch, you may be able to characterize the response space more naturally (with less dependence on explicit stitching splines, or on artificially inserted knots).
5. Liquidity-Fade and Liquidity-Explosion in practice: In practice the actual predictor ordinates across the manifest stretches will be too discrete for tracking the liquidity-fade and liquidity-explosion. Thus, it may be more appropriate to operate on predictor windows. If convenient and admissible, the predictor window boundaries may also coincide with the segment boundaries.

Stretch Partition/Isolation in Transition Splines

1. Definition: A given calibratable predictor ordinate/response realization space is called a span. The span is partitioned into stretches. Stretches can be either core stretches or transition stretches. Both the core stretches and the transition stretches are built from segments (within which the response values may be represented using basis splines). Core stretch are inferred to truthness and the smoothness signals, and the transition stretches provide the explicit bridge between the core stretches that may not be possible using the plain core stretch representations.
2. Information Patterns: With a higher unit, information propagation is associated with each sub-unit entities below. Across peer units, information exchange is materially similar in nature. Across higher units, information exchange may be more parsimonious (although it may still happen between lower entities belonging to the higher units).
3. Information Localization and Transmission: Intra-segment information propagation occurs through smoothness constraints such as C^k.
4. Stretch-Level Information Localization: In the spline case, this happens though boundary-condition delimitation/isolation (i.e., natural/financial/clamped boundary conditions based isolation is applicable to within a single stretch).
5. Stretch-Stretch Transmission: These are not bound by the equivalent isolation constraints, therefore the connecting/transition splines need to have a qualitatively different nature.
6. Transition/Connecting Splines: By definition, since they are the bridge between the stretches, they need to have greater degrees of freedom for a complete bridge.

Knot Insertion vs. Transition Splines

1. Equivalence: In some sense, they are equivalent in that inserting knots also attempts to complete the bridge. However, transition splines are more customizable, since the splines that flank the knots are assumed in the literature to be variants of the others.
2. Advantages on Knot Insertion: Remember that transition splines need $2k + 2$ basis function. Thus, for high k, you are stuck with higher-order polynomials (for e.g.), along with all the Runge's oscillations/instabilities that it brings. Suitable choice of knots may minimize this.
3. Advantage of Transition Spline: Knots are stretch response altering (via their C^k criteria), whereas transition splines enable each stretch to retain their character.

Overlapping Stretches

1. Premise: By definition, stretch fade-out and stretch explode axiomatizations imply predictor ordinate overlapping stretches.
2. Stretch Boundaries: Each stretch constituting an overlapping stretch needs to have its boundaries identified. What ***do*** not necessarily overlap are the smoothness constraints.
3. Overlapping Stretch – Problem Statement:
   - Predictor Ordinate Stretches overlap.
   - Stretches (and by implication, their predicate ranges) are contained/telescoped.
   - Smoothness constraints may not overlap, in which case they are posited to be distinct in each of the constituent stretches.
   - Truthness should be strictly telescopically contained/localized, i.e., there is a ***manifest measurement exclusivity*** to each stretch.
   - A consequence of this is that the inferred state response variable will be propagated, but not (necessarily) the smoothness criterion.

Index/Tenor Basis Swaps

Component Layout and Motivation

1. Basis Swap Market: Although Basis Swaps did exist even earlier (Tuck man and Porfirio (2003), Morini (2008)), post-crisis segmentation (attributable, among other things, to the preference towards receiving higher frequency payments) intensified these differentials (Mercurio (2009)).
2. Origins of Basis Swap Existence: In principle, these are expected to represent embedded duration counter-party credit risk. The "good" model should couple embedded credit risk with the sided flow dynamics (i.e., the credit quality of the counter-party that enters into the long/short side of the greater frequency leg, etc :)
3. The Discounting Curve: Challenges regarding the uniqueness in relation to the instrument choice for building the discount curve have been identified by Henrard (2007). The issues stem primarily from the uncollateralized nature of deposits and forwards, therefore, these are typically replaced by OIS/EONIA and Futures (Madigan (2008)).
    - Interest Rate Swap continues to be used for the discount curve calibration, as it possesses the following characteristics:
        - Par IRS'es are collateralized at inception.
        - Collateral margining may be applied over time.
        - IRS is the only liquidly available fix-float swap, and as such effectively implies just a single forward curve.
    - Convexity adjustment for extracting the rate from future/forward price => Since futures/forwards act effectively as a zero coupon bond, the transformation of price to the latent zero/forward rate requires a dynamical volatility based curve evolution model. Sophisticated, comprehensive approaches are available in literature (see for e.g., Kirikos and Novak (1997), Jackel and Kawai (2005), Brigo and Mercurio (2006), Piterbarg and Renedo (2006)); common practitioner

approaches, however, employ simpler approaches such as the Hull-White one-factor short-rate model (Hull and White (1990)).

4. Multi Curve vs. Forward Smoothness: Given that the discount curve and the forward curve are essentially distinct in the multi-curve latent state, the stringent demands that all forwards stay smooth (as in the single discount curve that covers all the basis curve scenarios) may be relaxed.
   - Forwards Implied in the Discount Curve => Since the forwards are used only for the "core" tenor pillars in the discount curve, only those forwards need to be smooth (e.g., 6M forwards). By discount curve construction this will typically be the case, as the forwards period will always straddle/span fully a single reset pillar.
5. Point- vs. Convolved-Measure State Transform:
   - Point-Measure transform refers to the one-to-one transform between a state measure at a predictor ordinate and its corresponding observation (e.g., discount factor from zero-coupon bond price observations). Since these may be expressed as straightforward transformations, the observation-state non-linearity may be easily accommodated.
   - Convolved measure-state transforms introduce what are effectively observation constraints across predictor ordinate/state response combinations. Non-linearity introduces complications, therefore usage of spline-based linearization constraints are highly effective.
6. Reset-Date Forward-Rate Pair Constraint in Discount Curve Building: The yM tenor (e.g., $yM \Rightarrow 6M$) may be extracted only at the reset start/end date (depending on the reset rate-rime axis label) from the discount curve, i.e., only the pair $\langle yM, Forward_{yM} \rangle$ makes sense. In other words, this is the only set of dates for which the information on forward rates is available. Splining may be an option at the other dates.
   - yM Tenor/DF Relationship => $PV_{yM} = \sum_{j=1}^{m} \Delta(j-1, j) F_{yM}(t_j) D_f(t_j)$. For PV_{yM} to be telescoped away into $PV_{yM} = D_f(t_0) - D_f(t_j)$, the requirements are: Period

Accrual End Date == Period Reset End Date == Period Pay Date. This is the main reason why the period dates are adjusted before the cash flows are rolled out.

7. <u>Alternative View: Discount Curve IS the yM Forward Curve</u>: To automatically ensure uniqueness and consistency of the latent state space, it may also be more restrictively imposed that the native yM Forward Curve be implied entirely off of the discount curve. Thus, the native yM Forward Curve may now be implied at all nodes, not just at the reset nodes as postulated earlier. This automatically eliminates the state basis between these measures; further, this is still not too restrictive in terms of the native yM Forward Curve smoothness for same reasons as before.
8. <u>Basis between the yM Forward Curve and the Discount Curve</u>: Given that basis constraints are of paramount consideration in other markets, why not look at the basis between discount curve and its native forward curve? This is because neither the latent state underpinning the forward curve or that underpinning the discount curve is entirely observable (unlike, say basis between a bond and the issuer's underlying CDS). Thus an extraneous observation model is necessary. By convention, the current practice achieves this by construction – the formulation mandates that the discount curve and the "discounting-native" forward curve be alternate quantification metrics of the same latent state.

Formulation

1. <u>Float-Float Swap Setup</u>: The phenomenology and flow details laid out in Figure 5 are based off of descriptions and details provided by ISDA (2000), Ametrano and Bianchetti (2009), Bianchetti (2009)). The two swap legs are:
    - The "known" or the "Reference" leg. Forwards of this leg come from the discount curve's IRS contracts, and 6M LIBOR/EURIBOR is the most common such tenor. We generalize this with a basis spread, i.e., the "effective" forward is $F_{6M} + S_{6M}$, where F_{6M} and S_{6M} stand for the corresponding forward and the spread.

- The "unknown" or the "Derived" leg with a tenor of xM. Forwards of this leg are computed from the corresponding basis market quotes. We generalize this with a basis spread, i.e., the "effective" forward is $F_{xM} + S_{xM}$, where F_{xM} and S_{xM} stand for the corresponding forward and the spread.

2. <u>Basic Formulation Setup</u>:
   - $PV_{xM} = \sum_{j=1}^{m} \Delta(j-1, j)\left[F_{xM}(t_j) + S_{xM}\right]D_f(t_j)$
   - $PV_{6M} = \sum_{a=1}^{b} \Delta(a-1, a)\left[F_{6M}(t_a) + S_{6M}\right]D_f(t_a)$
   - Equivalence of S_{xM} and S_{6M} => Since both S_{xM} and S_{6M} are additive, we work in a space that is essentially an adjusted forward rate space, with $F_{6M,Adj} \to F_{6M} + S_{6M}$ and $F_{xM,Adj} \to F_{xM} + S_{xM}$. While this is straightforward to accommodate in the case of $6M$, from a calibration point-of-view, we work off of a biased xM space, and re-adjust back after splining.
3. <u>Basis Swap Calibration Formulation</u>: $PV_{xM} = PV_{6M}$ implies that

$$\sum_{j>m_l}^{m} \Delta(j-1, j)F_{xM,Adj}(t_j)D_f(t_j) = \sum_{a=1}^{b} \Delta(a-1, a)F_{6M,Adj}(t_a)D_f(t_a) - \sum_{j=1}^{m_l} \Delta(j-1, j)F_{xM,Adj}(t_j)D_f(t_j)$$

   . For all but the left most basis swap, $m_l > 0$.
4. <u>Basis Swap Calibration Constraint Specification</u>:
   - Set $\aleph_m = \sum_{a=1}^{b} \Delta(a-1, a)F_{6M,Adj}(t_a)D_f(t_a) - \sum_{j=1}^{m_l} \Delta(j-1, j)F_{xM,Adj}(t_j)D_f(t_j)$. Notice that $\aleph_m$ maybe fully computed from before.
   - Recognize that $F_{xM,Adj}(t) = \sum_{i=1}^{n} \beta_i f_i(t)$.
   - Combine above to get the calibration constraint

$$\aleph_m = \sum_{i=1}^{n} \beta_i \left\{ \sum_{j>m_l}^{m} \Delta(j-1, j) f_i(t_j) D_f(t_j) \right\}.$$

5. <u>Reference/Derived Par Spread Relations</u>: For parity, $PV_{\mathrm{Re}\,ference} + DV01_{\mathrm{Re}\,ference} S_{\mathrm{Re}\,ference} + PV_{Derived} + DV01_{Derived} S_{Derived}$. Setting $S_{Derived} = 0$,

$S_{\mathrm{Re}\ ferecne} = -\dfrac{PV_{\mathrm{Re}\ ferecne} + PV_{Derived}}{DV01_{\mathrm{Re}\ ferecne}}$. Likewise, $S_{Derived} = -\dfrac{PV_{\mathrm{Re}\ ferecne} + PV_{Derived}}{DV01_{Derived}}$.

Remember that both $S_{\mathrm{Re}\ ferecne}$ and $S_{Derived}$ can be negative.

Multi-Stretch Merged Curve Construction

Motivation

1. Discount Curve composed of Forward Rate Stretches: The discount curve span may be viewed as being composed of overlapping/non-overlapping forward rate stretches, i.e., adjacent or otherwise 3M Tenor forward stretch, 6M Tenor forward stretch, etc: This visualization is a consequence of the representation of the “single discount curve latent state”, whose alternate/parallel quantification metrics are composed off of these stretches of forward rates that share the latent state space with the global discount curve.
2. Out-of-Native Stretch Arbitrage: If one seeks a forward rate outside these stretches for the given tenor/index combination, there can be no expectations of no-arbitrage, i.e., there will be a basis between the forward implied by this latent space quantification metric and the forward rate under consideration.
    - Likewise, if inside the stretch, there should be no implied basis, since the diver latent state is identical/fully correlated.
3. Merging/de-merging of the Latent State along the Predictor Ordinates: If you imagine the rates state space being characterized by a set of latent states (which may be highly correlated), each state may ideally be characterized by a quantification metric that is native to the state physical view. Thus, the unification of the sub-states in a stretch may be viewed as state-merging (i.e., one quantification metric may be inferred from another within a merged space via a trivial transformation).
4. Probit-based Latent State Merger Analysis: Given that the discount/forward latent states merge/de-merge, it might it particularly amenable to a common-factor probit (or even a logistic) analysis of the merger driver dynamics. The challenge would then be to link the driver dynamics to the maturity based predictor ordinate.

Merge Stretch Calibration

1. Cross-Stretch Calibration: Clearly the latent state span characterized by multiple stretches will in turn be composed of latent state merge sub-stretches. The merged stretch may be followed by de-merged stretch, etc:
2. Calibration Challenges:
    - What would be most optimal cross-representation inside the merge sub-stretch (i.e., the state representation needs to be smooth for both the discount factor latent state as well as the forward curve latent state)?
    - On the other hand in the solitary segment sub-stretch, you may have more representation freedom, but may still need to carry over the smoothness constraints from the merged sub-stretch. How can this be done? Can the transition spline treatment above be effectively employed here? In other words, what would be appropriate transition zone applicable to the sub-stretch?

Spline-Based Credit Curve Calibration

1. Overview: Andersen (2003) has made an initial effort in this regard.

Inference-Based Curve Construction

Curve Smoothing in Finance

1. Unconstrained Curve Smoothing:
    - Applicable primarily for rates/semi-liquid FX curves. Smoothing can be done here without constraints.
    - Smoothing may also be applicable to the quotes for a given instrument across several days.
    - Smoothing may also be applied over a single day curve – particularly to model the switch over from instrument to instrument (e.g., between EDF and Swaps).
2. Constrained Curve Smoothing: Applicable, for e.g., to the case of a hazard curve. The smoothing basis functions/weights combination must guarantee, from a formulation PoV, that the implied hazard rate is always greater than zero.
3. Liquidity Based Weighted Signal Smoothing:
    - Fidelity at the "liquid bonds" / benchmark bond nodes
    - Lower fidelity penalty, but higher smoothness penalty for the less liquid bonds
    - Penalty measure is calculated off of the relative liquidity ranking measure (for e.g., TRACE)
4. Non Bayesian Liquidity Based Smoothing:
    - Liquidity indicator serves as a roughness/fidelity magnifier/dampener
    - Also need to penalize for over-parameterized fits using AIC/BIC (also CV/GCV – given that this is essentially a frequentist case).
    - These can be applied not just for bonds, but also CDS, rates, FX – even less liquid ones.
5. Bayesian Extension to the above: Any parametrically specified distribution needs to evolve using a hyper-prior, and the Wahba parametric Bayesian priors need evolving too.

6. Nodal Jacobian/Sensitivity Impact: As always study the impact on the locality of the perturbation, as well as the ease of Jacobian estimation – esp. if the calibration needs to occur through MCMC, non-linear optimization etc:
7. Mixtures of splines and smoothness penalties: As always estimate the impact on monotonicity, convexity, shape preservation etc: - the category item checks in Goodman's paper.
8. Knot Selection Tips: Need some tips in both situations – frequentist and Bayesian.
9. Suggestion on the locally adaptive Parametric Form: Examine the knot-to-knot smoothness and penalty by using additional locally adaptive microstructure parameters and their implications.
10. Goodman and Eilers/Marx Talking Point Issues: Criterion check for these specific "goodness" checks.

Bayesian Curve Calibration

1. Bayesian based past knowledge incorporation of survival probabilities: Given that the prior's, the posterior's and the likelihood's are all probabilities, perhaps the best starting point is for applying it to the problem of updating the survival probabilities and recovery rates based on price observations.
2. Curve Updating techniques: Need grand new formulation techniques that are based on AD and Bayesian methodologies as part of the curve updating strategies based upon individual incoming observations and their strength signals.
3. Curve Construction off of hard/soft signals: Hard Signals are typically the truthness signals. Typically reduce to one calibration parameter per hard observation, and they include the following:
    - Actual observations => Weight independent true truthness signals
    - Weights => Potentially indicative of the truthness hard signal strength

   Soft signals are essentially signals extracted from inference schemes. Again, typically reduce to one calibration parameter per soft inference unit, and they include the following:

- Smoothness signals => Continuity, first, second, and higher-order derivatives match – one parameter per match.
- Bayesian update metrics => Inferred using Bayesian methodologies such as maximum likelihood estimates, variance minimization, and error minimization techniques.

4. No-arbitrage hard signals: Simply indicates that ***the*** given hard observation is out of bounds and irreconcilable (i.e., no solution can be found) within the axiomatic inference space dictated by:
    i. The parameter sequence implied by the other set of hard signals.
    ii. The model axiom schemes.
    iii. The inference rules.
    - Directionality "bias" is inherent in calibration (e.g., left to right, ordered sequence set, etc:) – this simplifies the problem space significantly. Therefore, the same directional bias also exists in the calibration nodal sequence.
5. Parameter Space Explosion: Generally not a problem as long as it is segment-localized (in matrix parlance, as long the transition matrix is tri-diagonal, or close to it), i.e., local information discovery does not affect far away nodes/segments.
    - Also maybe able to use optimization techniques to trim them.
6. Live Calibrated Parameter Updating: Use automatic differentiation to:
    - Estimate parametric Jacobians (or sub-coefficient micro-Jacobians) to the observed product measures.
    - Re-adjust the shifts using the hard-signal strength.
    - Update the parameters from the calculated shifts.
    - Re-construct the curve ever so periodically (for a full re-build, as opposed to the incrementals).
    - Remember that AD based parametric updates break smoothness (including continuity as Bayesian MLE's) – so use a tolerance in the shift if this is acceptable.
7. Causality Bayesian Network DAG For Credit Curve Building: See Figure 1.

- DAG searches are not really needed, since here they maybe formulated conceptually/axiomatically, as opposed to being established through a search mechanism.

8. Financial First Principles SKU: Following concepts are the core components that can be used to create the curve construction SKU:
    - Time Value of Money.
    - Latent Default Indicator.
    - Recovery on Default.
    - Imbalance premium/discount (for FX, Basis Swaps, etc.)
9. Financial Signal Analysis: Need special analysis techniques to pick out "event trends" from "concept jumps", even for highly liquid instruments.
    - Liquidity-based Signal Extraction =>
        - Identify a liquidity metric
        - Imply the "perfect liquidity" – the point at which there is no premium
        - Compute the liquidity metric for each security
        - Regress (or conceptually determine, or fit) the bid-ask spread to inverse liquidity (remember that even benchmarks only have finite liquidity, not infinite) for each security
        - Try to slap in a secular "event premium" across all the instruments, over and above liquidity
10. Systemic Finance Variables Evolution: Given that every measurement is uncertain to within bounds, it stands to reason that every distribution is also a true distribution (to within the tolerance provided by the corresponding sufficient statistics, and over a finite observation window) of the technical state of the world (i.e., technical = fundamental + a bias).
11. Technical to Fundamental Bias Estimation: This should result from the flow of the information. Non-technical/Fundamental may possibly be estimated using a bias correction applied to the technical signal – Bayesian/frequentist techniques may be of value here.
    - Proxy for non-technical behavior => Identify the non-market proxies for the fundamental drivers, and estimate market drivers as possibly lagging indices.

12. Bayesian Decomposition of Technical Signals: In general, the signal core drivers are limited (like systemic/idiosyncratic factors – alternatively, the latent state quantification metric), but the product specific manifest measures are more varied. Bayesian frameworks well suited for these.
13. Financial Stretch Identification: Bayesian classification techniques can be readily adapted for these purposes – in fact, with abundance of data, these techniques are very appropriate now.

Sequential Curve Estimation

1. Calibration Framework Drivers: Calibration is considered to occur FOR a hidden state $\vec{S}$, which is quantified using the quantification metric $\vec{X}$. $\vec{X}$ is estimated from the manifest measure $\vec{M}$.
2. Product-Measure Point-of-View: From the Dempster-Shaefer/Kalman Filter/Linear Quadratic Estimator point-of-view, the Kalman $\vec{H}$ matrix probabilistically transforms the hidden state quantification metric to an observation measure, e.g., the latent forward rate manifests itself through the swap rate.
3. Segment/Span Nomenclature vs. Curve Calibrator Nomenclature: Call the Curve Calibrator the Dempster-Shaefer Calibrator. Under this:
   - LSQM (Latent State Quantification Metric) => Elastic Variate
   - State Dimensions (Tenor Axis, X/Y Axis of predictors) => Inelastic Variates
   - Thus, the predictors are inelastic, and the responses are elastic.
4. Linearization of LSQM over the predictor axes: The Kalman $\vec{H}$ observation transformer should just linearize $\vec{M}$ onto the space of $\vec{X}$ over the predictor dimensions. Non-linearity of $\vec{X}$ over the predictors is handled through basis splines.
5. Hidden State Evolution vs. Hidden State Representation: The Kalman $\vec{H}$ matrix is more of a state modeling and state representation matrix (i.e., the update part that is fully local to the current time slice) that already brings in the manifest measure ⇔ LSQM transformation model.

6. <u>The Curve Builder $\vec{H}$ matrix</u>: Due to the above, the curve builder $\vec{H}$ matrix needs to accommodate the 2 possible uncertainties:
   - Uncertainty in the manifest measure
   - Uncertainty in the manifest measure ⇔ LSQM transformation model. If this transformation is non-parametric, then treat it as certain/deterministic. If it is parametric, then use MLE/MAP to the handle the parameter estimation.
7. <u>UKF Techniques applied to evolve the Curve Builder $\vec{H}$ matrix</u>: Potential non-linearity in the curve builder $\vec{H}$ may be handled using the Jacobian EKF and/or the sigma-point UKF schemes.
8. <u>The Curve Builder $\vec{F}$ matrix</u>: The Curve builder $\vec{F}$ Matrix dictates the evolution from t_i to t_{i+1} as $LSQM_{i+1} = \vec{F} \times LSQM_i$. This should be explicitly posited/formulated. Again, use splining to linearize.
9. <u>Financial Noise Covariance Estimation</u>: May be able to extraneously determine these covariance independent of the state evolution model (if not, we may have to rely on techniques such as ALS (Rajamani (2007), Rajamani and Rawlings (2009)).

Appendix A: Some Trivial Analytical Bond Math Results

1. Price when Yield Equals Coupon: Given the following:
   - Annualized Coupon => r
   - Payment frequency => f
   - Per period yield => y
   - Per Period Coupon Payment => $c = \frac{r}{f}$
   - Number of coupon periods to maturity => n
   - $PV = \sum_{m=1}^{n} \frac{(r/f)}{(1+y)^m} + \frac{1}{(1+y)^n} = \sum_{m=1}^{n} \frac{c}{(1+y)^m} + \frac{1}{(1+y)^n} = cd\frac{1-d^n}{1-d} + d^n$ where $d = \frac{1}{1+y}$.
   - Now, if you are just past a coupon pay (so that clean == dirty), and if $PV = 1$, then we get $1 = cd\frac{1-d^n}{1-d} + d^n \Rightarrow d = \frac{1}{1+c} \Rightarrow y = c$. QED.
2. Par Yield Dirty Price at a non-coupon Date: If ξ is the accrual fraction corresponding to the accruing period, then $PV_{Dirty} = \frac{\xi c}{(1+y)^{\xi}} + \frac{c}{(1+y)^{\xi+1}} + \ldots + \frac{c}{(1+y)^{\xi+n}} + \frac{1}{(1+y)^{\xi+n}}$

   => $PV_{Dirty} = \frac{\xi c}{(1+y)^{\xi}} + \frac{1}{(1+y)^{\xi}}\left[\sum_{m=1}^{n} \frac{c}{(1+y)^m} + \frac{1}{(1+y)^n}\right] = \frac{1+\xi c}{(1+y)^{\xi}}$.

Appendix B: Per-trade Risk Isolation Components

1. Underlier Security Price Market Risk
2. Discount Factor Risk
3. Forward Rate Risk
4. Currency/FX Risk
5. Basis Risk (on any Risk Factor)
6. Funding Risk
7. Collateral Risk
8. Counter-party Risk

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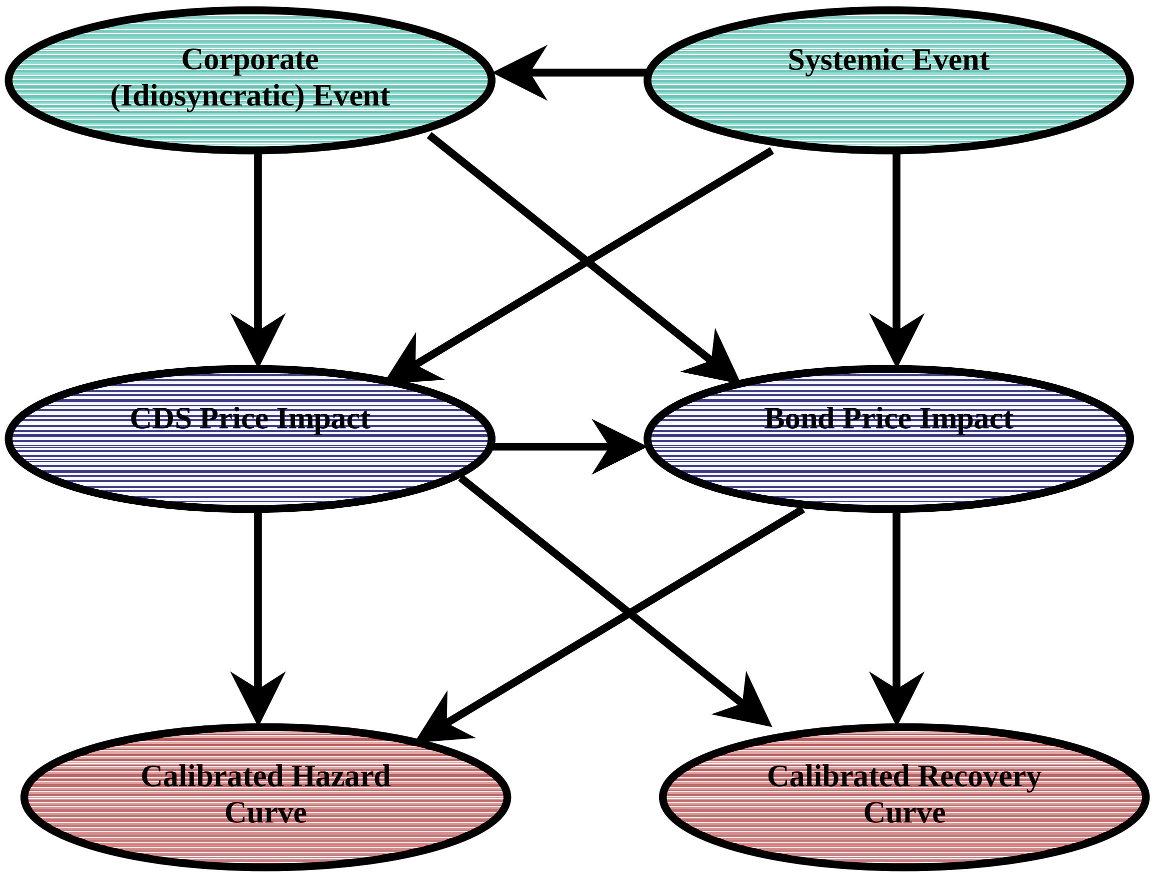


Figure 1: Causality Bayesian Network DAG For Credit Curve Building

Figure 2: Transition Splines – Low Width Transition Stretch

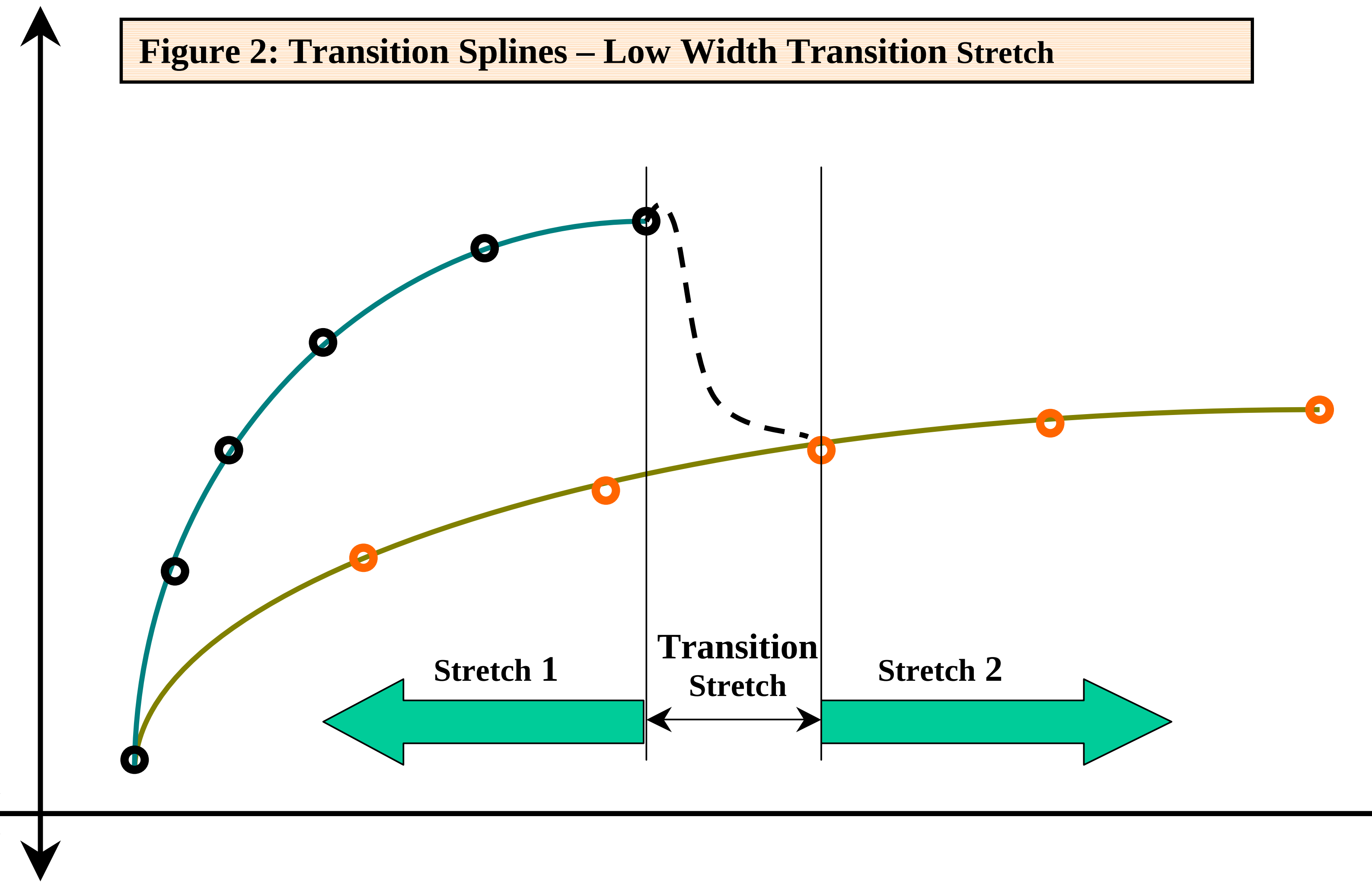


Figure 3: Transition Splines – High Width Transition Stretch

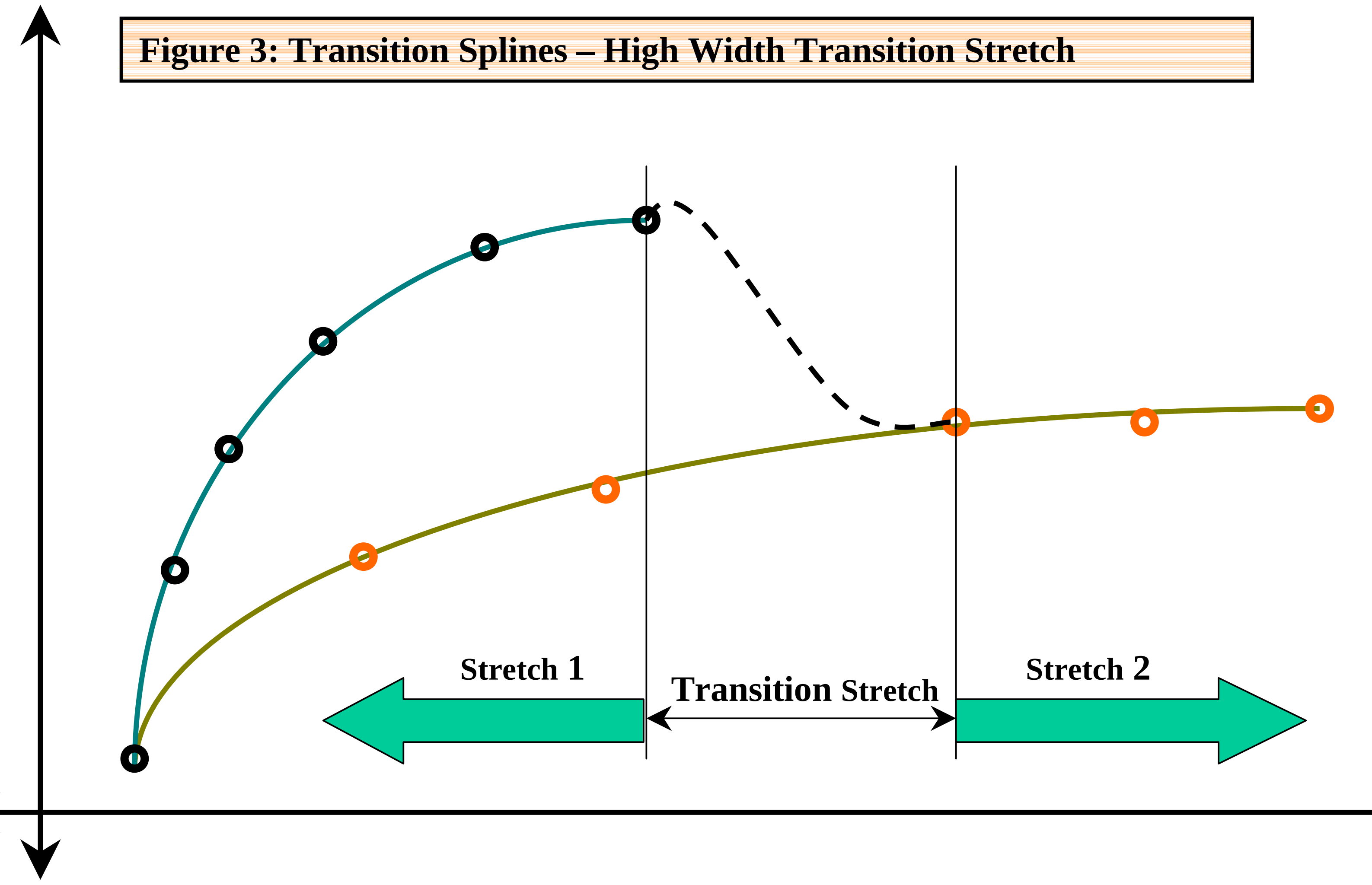


Figure 4: Transition Splines – Segment <-> Stretch Layout

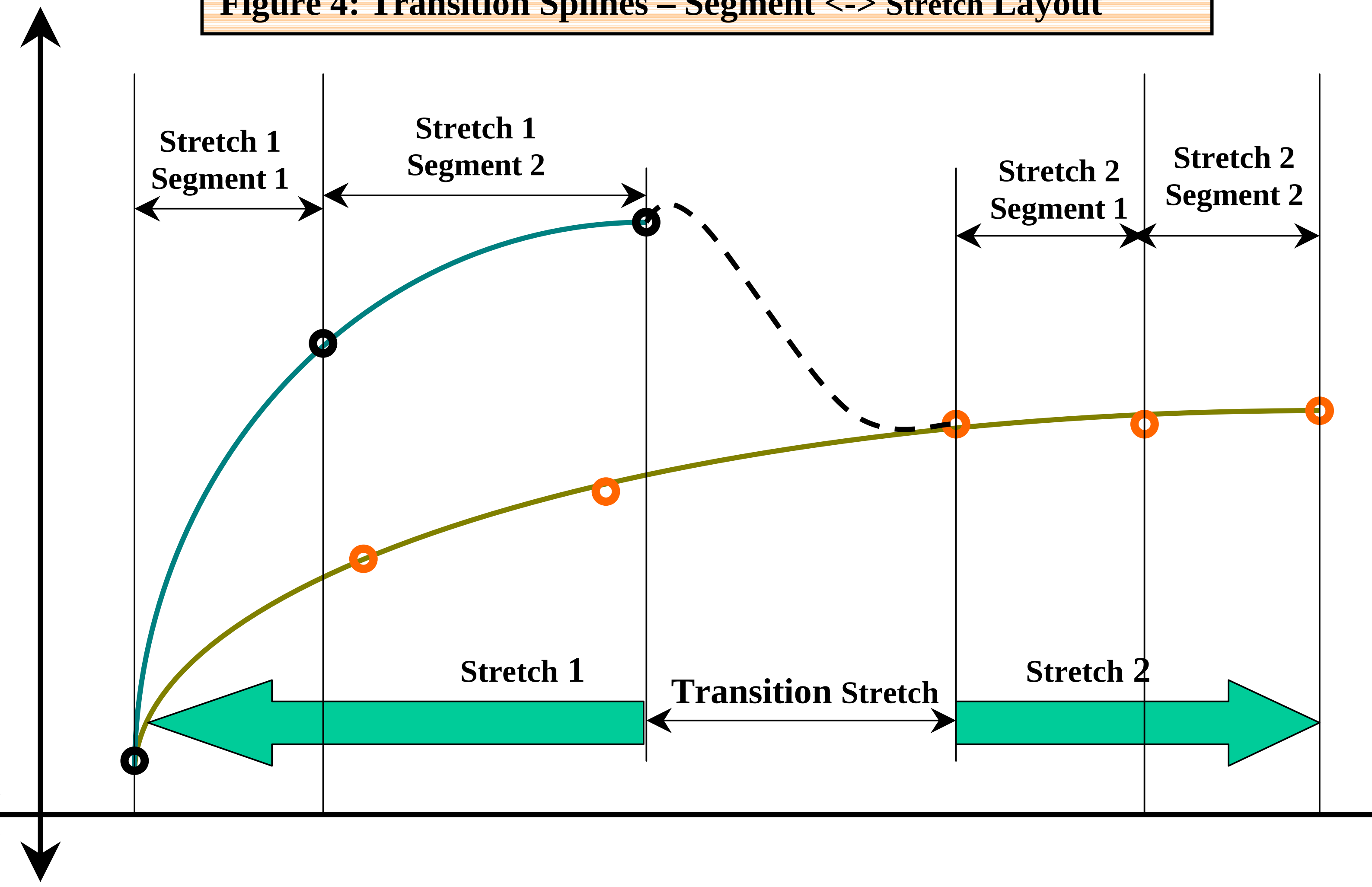
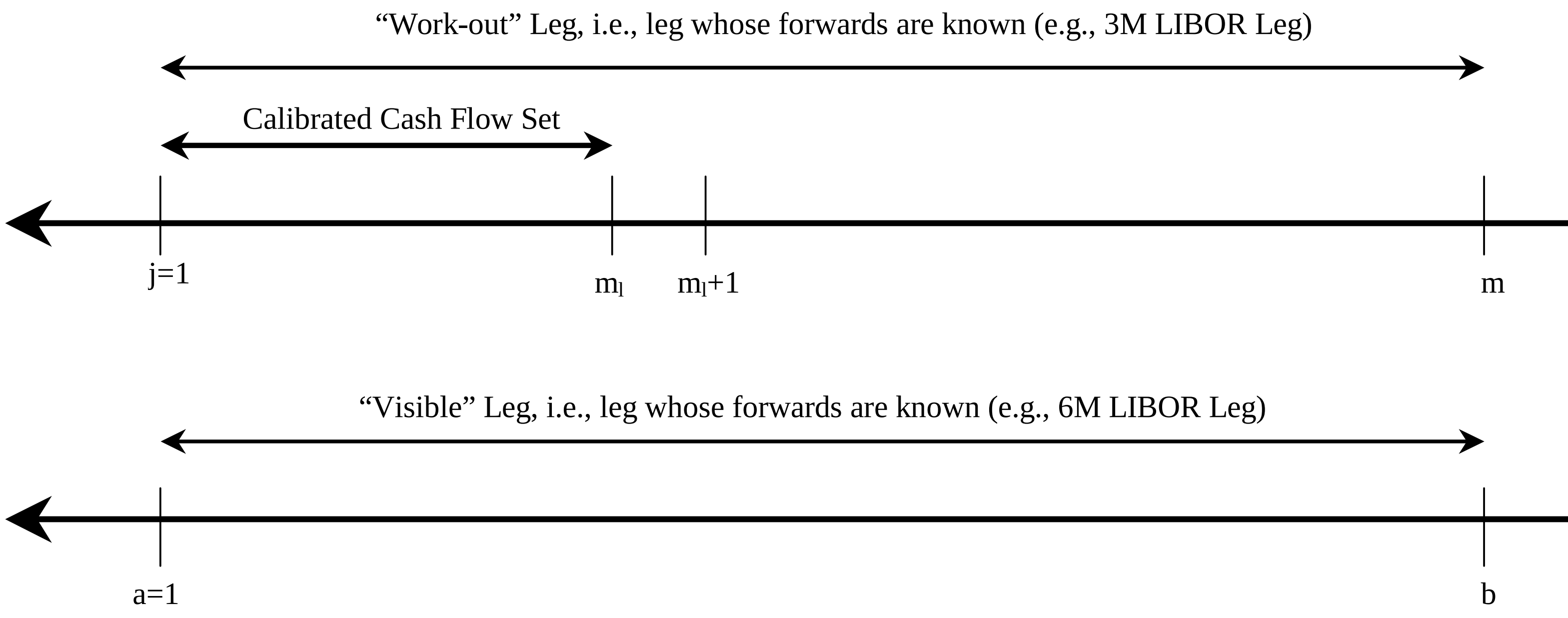


Figure 5: Float-Float Swap Set-up



“Work-out” Leg, i.e., leg whose forwards are known (e.g., 3M LIBOR Leg)
Calibrated Cash Flow Set
j=1
m_l
m_l+1
m
“Visible” Leg, i.e., leg whose forwards are known (e.g., 6M LIBOR Leg)
a=1
b

Curve Builder Software Components

The Curve Builder Software Components are implemented across 6 core functional packages, 3 sample packages, and 2 regression test packages.

The core functional packages are:

- Latent State Representation Package
- Latent Curve State Package
- Latent State Estimation Package
- Latent State Creation package
- Analytics Definition Package
- Rates Analytics Package
- Rates Sample Package
- Credit Sample Package
- Bloomberg Sample Package
- Curve Regression Package
- Curve Jacobian Regression Package

Latent State Representation Package (org.drip.state.representation)

The latent state representation package implements the latent state, the quantification metric/manifest measure, its labels, the merge stretch and its manager. It contains the following classes/interfaces:

1. LatentStatelLabel: LatentStateLabel is the interface that contains the labels inside the sub-stretch of the alternate state. The functionality its derivations implement provide fully qualified label names and their matches.
2. LatentStateMergeSubStretch: LatentStateMergeSubStretch implements merged stretch that is common to multiple latent states. It is identified by the start/end date

predictor ordinates, and the Latent State Label. Its methods provide the following functionality:

- Identify if the specified predictor ordinate belongs to the sub stretch
- Shift that sub stretch start/end
- Identify if the this overlaps the supplied sub stretch, and coalesce them if possible
- Retrieve the label, start, and end

3. MergeSubStretchManager: MergeSubStretchManager manages the different discount-forward merge stretches. It provides functionality to create, expand, or contract the merge stretches.
4. LatentStateMetricMeasure: LatentStateMetricMeasure holds the latent state that is estimated, its quantification metric, and the corresponding product manifest measure, and its value that it is estimated off of during the calibration run.
5. LatentState: LatentState exposes the functionality to manipulate the hidden Variable's Latent State. Specifically it exports functions to:
   - Retrieve the Array of the LatentStateMetricMeasure
   - Produce node shifted, parallel shifted, and custom manifest-measure tweaked variants of the Latent State
   - Produce parallel shifted and custom quantification metric tweaked variants of the Latent State

Latent State Estimator Package (org.drip.state.estimator)

The latent state estimator package provides functionality to estimate the latent state, local/global state construction controls, constraint representation, and linear/non-linear calibrator routines. It contains the following classes/interfaces:

1. StretchRepresentationSpec: StretchRepresentationSpec carries the calibration instruments and the corresponding calibration parameter set in LSMM instances. Together, these inputs are used for constructing an entire latent state stretch. StretchRepresentationSpec exports the following functionality:
   - Alternate ways of constructing custom Stretch representations

- Retrieve indexed instrument/LSMM
- Retrieve the full set calibratable instrument/LSMM

2. <u>PredictorResponseWeightConstraint</u>: PredictorResponseWeightConstraint holds the Linearized Constraints (and, optionally, their quote sensitivities) necessary needed for the Linear Calibration. Linearized Constraints are expressed as $C_j = \sum_i W_i y(x_{ij})$ where x_{ij} is the predictor ordinate at node i, y is the response, W_i is the weight applied for the Response i, and C_j is the value of constraint j. The function can either be univariate function, or weighted spline basis set. To this end, it implements the following functionality:
   - Update/Retrieve Predictor/Response Weights and their Quote Sensitivities
   - Update/Retrieve Predictor/Response Constraint Values and their Quote Sensitivities
   - Display the contents of PredictorResponseWeightConstraint
3. <u>SmoothingCurveStretchParams</u>: SmoothingCurveStretchParams contains the Parameters needed to hold the Stretch. It provides functionality to:
   - The Stretch Best fit Response and the corresponding Quote Sensitivity
   - The Calibration Detail and the Curve Smoothening Quantification Metric
   - The Segment Builder Parameters
4. <u>GlobalCurveControlParams</u>: GlobalControlCurveParams enhances the SmoothingCurveStretchParams to produce globally customized curve smoothing. Currently, GlobalControlCurveParams uses custom boundary setting and spline details to implement the global smoothing pass.
5. <u>LocalCurveControlParams</u>: LocalControlCurveParams enhances the SmoothingCurveStretchParams to produce locally customized curve smoothing. Flags implemented by LocalControlCurveParams control the following:
   - The C1 generator scheme to be used
   - Whether to eliminate spurious extrema
   - Whether or not to apply monotone filtering.

6. <u>CurveStretch</u>: CurveStretch expands the regular Multi-Segment Stretch to aid the calibration of Boot-strapped Instruments. In particular, CurveStretch implements the following functions that are used at different stages of curve construction sequence:
    - Mark the Range of the "built" Segments
    - Clear the built range mark to signal the start of a fresh calibration run
    - Indicate if the specified Predictor Ordinate is inside the "Built" Range
    - Retrieve the MergeSubStretchManager
7. <u>RatesSegmentSequenceBuilder</u>: RatesSegmentSequenceBuilder holds the logic behind building the bootstrap segments contained in the given Stretch. It extends the SegmentSequenceBuilder interface by implementing/customizing the calibration of the starting as well as the subsequent segments.
8. <u>LinearCurveCalibrator</u>: LinearCurveCalibrator creates the discount curve span from the instrument cash flows. The span construction may be customized using specific settings provided in GlobalControlCurveParams.
9. <u>NonlinearCurveCalibrator</u>: NonlinearCurveCalibrator calibrates the discount and credit/hazard curves from the components and their quotes. NonlinearCurveCalibrator employs a set of techniques for achieving this calibration.
    - It bootstraps the nodes in sequence to calibrate the curve
    - In conjunction with splining estimation techniques, it may also be used to perform dual sweep calibration. The inner sweep achieves the calibration of the segment spline parameters, while the outer sweep calibrates iteratively for the targeted boundary conditions
    - It may also be used to custom calibrate a single Interest Rate/Hazard Rate Node from the corresponding Component
    - CurveCalibrator bootstraps/cooks both discount curves and credit curves
10. <u>RatesCurveScenarioGenerator</u>: RatesCurveScenarioGenerator uses the interest rate calibration instruments along with the component calibrator to produce scenario interest rate curves. RatesCurveScenarioGenerator typically first constructs the actual curve calibrator instance to localize the intelligence around curve construction. It then uses this curve calibrator instance to build individual curves or the sequence of node bumped scenario curves. The curves in the set may be an array, or tenor-keyed.

11. CreditCurveScenarioGenerator: CreditCurveScenarioGenerator uses the hazard rate calibration instruments along with the component calibrator to produce scenario hazard rate curves. CreditCurveScenarioGenerator typically first constructs the actual curve calibrator instance to localize the intelligence around curve construction. It then uses this curve calibrator instance to build individual curves or the sequence of node bumped scenario curves. The curves in the set may be an array, or tenor-keyed.

Latent Curve State Package (org.drip.curve.state)

The latent curve state package provides implementations of latent state representations of discount curve, forward curve, zero curve, credit curve, FX Basis curve, and FX forward curve. It contains the following classes/interfaces:

1. DiscountFactorDiscountCurve: DiscountFactorDiscountCurve manages the Discounting Latent State, using the Discount Factor as the State Response Representation. It exports the following functionality:
    - Compute the discount factor, forward rate, or the zero rate from the Discount Factor Latent State
    - Create a ForwardRateEstimator instance for the given Index
    - Retrieve Array of the Calibration Components and their LatentStateMetricMeasure's
    - Retrieve the Curve Construction Input Set
    - Compute the Jacobian of the Discount Factor Latent State to the input Quote
    - Synthesize scenario Latent State by parallel shifting/custom tweaking the quantification metric
    - Synthesize scenario Latent State by parallel/custom shifting/custom tweaking the manifest measure
    - Serialize into and de-serialize out of byte array
2. NonlinearDiscountFactorDiscountCurve: NonlinearDiscountFactorDiscountCurve manages the Discounting Latent State, using the Forward Rate as the State Response Representation. It exports the following functionality:

- Boot Methods - Set/Bump Specific Node Quantification Metric, or Set Flat Value
- Boot Calibration - Initialize Run, Compute Calibration Metric
- Compute the discount factor, forward rate, or the zero rate from the Forward Rate Latent State
- Create a ForwardRateEstimator instance for the given Index
- Retrieve Array of the Calibration Components and their LatentStateMetricMeasure's
- Retrieve the Curve Construction Input Set
- Compute the Jacobian of the Discount Factor Latent State to the input Quote
- Synthesize scenario Latent State by parallel shifting/custom tweaking the quantification metric
- Synthesize scenario Latent State by parallel/custom shifting/custom tweaking the manifest measure
- Serialize into and de-serialize out of byte array

3. ZeroRateDiscountCurve: ZeroRateDiscountCurve manages the Discounting Latent State, using the Zero Rate as the State Response Representation. It exports the following functionality:
   - Compute the discount factor, forward rate, or the zero rate from the Zero Rate Latent State
   - Create a ForwardRateEstimator instance for the given Index
   - Retrieve Array of the Calibration Components and their LatentStateMetricMeasure's
   - Retrieve the Curve Construction Input Set
   - Compute the Jacobian of the Discount Factor Latent State to the input Quote
   - Synthesize scenario Latent State by parallel shifting/custom tweaking the quantification metric
   - Synthesize scenario Latent State by parallel/custom shifting/custom tweaking the manifest measure
   - Serialize into and de-serialize out of byte array

4. <u>DerivedZeroRate</u>: DerivedZeroRate implements the delegated ZeroCurve functionality. Beyond discount factor/zero rate computation at specific cash pay nodes, all other functions are delegated to the embedded discount curve.
5. <u>FlatForwardDiscountCurve</u>: FlatForwardDiscountCurve manages the Discounting Latent State, using the Forward Rate as the State Response Representation. It exports the following functionality:
   - Boot Methods - Set/Bump Specific Node Quantification Metric, or Set Flat Value
   - Boot Calibration - Initialize Run, Compute Calibration Metric
   - Compute the discount factor, forward rate, or the zero rate from the Forward Rate Latent State
   - Create a ForwardRateEstimator instance for the given Index
   - Retrieve Array of the Calibration Components and their LatentStateMetricMeasure's
   - Retrieve the Curve Construction Input Set
   - Compute the Jacobian of the Discount Factor Latent State to the input Quote
   - Synthesize scenario Latent State by parallel shifting/custom tweaking the quantification metric
   - Synthesize scenario Latent State by parallel/custom shifting/custom tweaking the manifest measure
   - Serialize into and de-serialize out of byte array
6. <u>BasisSplineForwardRate</u>: BasisSplineForwardRate manages the Forward Latent State, using the Forward Rate as the State Response Representation. It exports the following functionality:
   - Calculate implied forward rate / implied forward rate Jacobian
   - Serialize into and de-serialize out of byte arrays
7. <u>ForwardHazardCreditCurve</u>: ForwardHazardCreditCurve manages the Survival Latent State, using the Hazard Rate as the State Response Representation. It exports the following functionality:
   - Boot Methods - Set/Bump Specific Node Quantification Metric, or Set Flat Value
   - Boot Calibration - Initialize Run, Compute Calibration Metric

- Compute the survival probability, recovery rate, or the hazard rate from the Hazard Rate Latent State
- Retrieve Array of the Calibration Components and their LatentStateMetricMeasure's
- Retrieve the Curve Construction Input Set
- Synthesize scenario Latent State by parallel shifting/custom tweaking the quantification metric
- Synthesize scenario Latent State by parallel/custom shifting/custom tweaking the manifest measure
- Serialize into and de-serialize out of byte array

8. DerivedFXForward: DerivedFXForward manages the constant forward based FX Forward Curve holder object. It exports the following functionality:
    - Extract currency, currency pair, spot epoch and spot FX
    - Compute Zero/boot-strap Basis, as well as boot-strap basis DC
    - Compute the spot implied rate/implied rate nodes
    - Retrieve Array of the Calibration Components and their LatentStateMetricMeasure's
    - Retrieve the Curve Construction Input Set
    - Synthesize scenario Latent State by parallel shifting/custom tweaking the quantification metric
    - Synthesize scenario Latent State by parallel/custom shifting/custom tweaking the manifest measure
    - Serialize into and de-serialize out of byte array
9. DerivedFXBasis: DerivedFXBasis manages the constant forward basis based FX Basis Curve holder object. It exports the following functionality:
    - Extract currency, currency pair, spot epoch, spot FX, and whether the basis is boot-strapped
    - Compute the FX Forward Array
    - Retrieve Array of the Calibration Components and their LatentStateMetricMeasure's

- Retrieve the Curve Construction Input Set
- Synthesize scenario Latent State by parallel shifting/custom tweaking the quantification metric
- Synthesize scenario Latent State by parallel/custom shifting/custom tweaking the manifest measure
- Serialize into and de-serialize out of byte array

Latent State Creator Package (org.drip.state.creator)

The latent curve state package provides implementations of the constructor factories that create discount curve, forward curve, zero curve, credit curve, FX Basis curve, and FX forward curve. It contains the following classes/interfaces:

1. DiscountCurveBuilder: This class contains the builder functions that construct the discount curve (comprising both the rates and the discount factors) instance. It contains static functions that build different types of discount curve from 3 major types of inputs:
   - From a variety of ordered DF-sensitive calibration instruments and their quotes
   - From an array of ordered discount factors
   - From a serialized byte stream of the discount curve instance
2. ZeroCurveBuilder: This class contains the builder functions that construct the zero curve instance. It contains static functions that build different types of zero curve from 2 major types of inputs:
   - From a source discount curve, a set of coupon periods, and the Zero Bump
   - From a serialized byte stream of the Zero curve instance
3. CreditCurveBuilder: This class contains the builder functions that construct the credit curve (comprising both survival and recovery) instance. It contains static functions that build different types of credit curve from 3 major types of inputs:
   - From a variety of ordered credit-sensitive calibration instruments and their quotes
   - From an array of ordered survival probabilities

- From a serialized byte stream of the credit curve instance

4. FXForwardCurveBuilder: This class contains the baseline FX Forward curve builder object. It contains static functions that build FX Forward curves from the 3 major inputs:
    - An ordered array of Forward FX
    - An ordered array of Forward Basis Points
    - A byte Stream representing the serialized instance of the FXForwardCurve
5. FXBasisCurveBuilder: This class contains the baseline FX Basis curve builder object. It contains static functions that build FX Basis curves from the 3 major inputs:
    - An ordered array of Forward FX
    - An ordered array of Forward Basis Points
    - A byte Stream representing the serialized instance of the FXBasisCurve

Analytics Definition Package (org.drip.analytics.definition)

The analytics definition package provides definitions of the generic curve, discount curve, forward curve, zero curve, credit curve, FX Basis curve, and FX forward curve, turns list, and their construction inputs. It contains the following classes/interfaces:

1. CurveConstructonInputSet: CurveConstructionInputSet interface contains the Parameters needed for the Curve Calibration/Estimation. It's methods expose access to the following:
    - Calibration Valuation Parameters
    - Calibration Quoting Parameters
    - Array of Calibration Instruments
    - Map of Calibration Quotes
    - Map of Calibration Measures
    - Double Map of the Date/Index Fixings
2. CurveSpanConstructionInput: CurveSpanConstructionInput contains the Parameters needed for the Curve Calibration/Estimation. It contains the following:

- Calibration Valuation Parameters
- Calibration Quoting Parameters
- Calibration Market Parameters
- Calibration Pricing Parameters
- Array of Calibration Stretch Representation
- Map of Calibration Quotes
- Map of Calibration Measures
- Double Map of the Date/Index Fixings
- Additional functions provide for retrieval of the above and specific instrument quotes. Derived Classes implement Targeted Curve Calibrators.

3. <u>ShapePreservingCCIS</u>: ShapePreservingCCIS extends the CurveSpanConstructionInput Instance. Additionally, it exposes the Shape Preserving Linear Curve Calibrator.
4. <u>BootCurveConstructionInput</u>: BootCurveConstructionInput contains the Parameters needed for the Curve Calibration/Estimation. It contains the following:
   - Calibration Valuation Parameters
   - Calibration Quoting Parameters
   - Array of Calibration Instruments
   - Map of Calibration Quotes
   - Map of Calibration Measures
   - Double Map of the Date/Index Fixings
5. <u>Curve</u>: Curve extends the Latent State to abstract the functionality required among all financial curve. It exposes the following functionality:
   - Set the Epoch and the Identifiers
   - Set up/retrieve the Calibration Inputs
   - Retrieve the Latent State Metric Measures
6. <u>CreditCurve</u>: CreditCurve is the stub for the survival curve functionality. It extends the Curve object by exposing the following functions:
   - Set of curve and market identifiers
   - Recovery to a specific date/tenor, and effective recovery between a date interval

- Hazard Rate to a specific date/tenor, and effective hazard rate between a date interval
- Survival to a specific date/tenor, and effective survival between a date interval
- Set/unset date of specific default
- Generate scenario curves from the base credit curve (flat/parallel/custom)
- Set/unset the Curve Construction Inputs, Latent State, and the Manifest Metrics
- Serialization/De-serialization to and from Byte Arrays

7. ExplicitBootCurve: In ExplicitBootCurve, the segment boundaries explicitly line up with the instrument maturity boundaries. This feature is exploited in building a boot-strappable curve. Functionality is provides set the Latent State at the Explicit Node, adjust the Latent State at the given Node, or set a common Flat Value across all Nodes.
8. ExplicitBootCreditCurve: ExplicitBootCreditCurve exposes the functionality associated with the bootstrapped Credit Curve.
9. FXForwardCurve: FXForwardCurve implements the curve representing the FXForward nodes. It extends the Curve class, and exposes the following functionality:
    - Retrieve the spot parameters (FX Spot, Spot Date, and the currency pair)
    - Calculate the Zero set of FX Basis/Zero Rate nodes corresponding to each basis node
    - Bootstrap basis points/discount curves corresponding to the FXForward node set
    - Imply the zero rate to a given date from the FXForward curve
10. FXBasisCurve: FXBasisCurve implements the curve representing the FXBasis nodes. It extends the Curve class, and exposes the following functionality:
    - Retrieve the spot parameters (FX Spot, Spot Date, and the currency pair)
    - Indicate if the basis has been bootstrapped
    - Calculate the Complete set of FX Forward corresponding to each basis node

Rates Analytics Package (org.drip.analytics.rates)

The rates analytics package provides definitions of the discount curve, the forward curve, the zero curve, the discount factor and the forward rate estimators, the turns list, and their construction inputs. It contains the following classes/interfaces:

1. DiscountFactorEstimator: DiscountFactorEstimator is the interface that exposes the calculation of the Discount Factor for a specific Sovereign/Jurisdiction Span. It exposes the following functionality:
    - Curve Epoch Date
    - Discount Factor Target/Effective Variants - to Specified Julian Dates and/or Tenors
    - Forward Rate Target/Effective Variants - to Specified Julian Dates and/or Tenors
    - Zero Rate Target/Effective Variants - to Specified Julian Dates and/or Tenors
    - LIBOR Rate and LIBOR01 Target/Effective Variants - to Specified Julian Dates and/or Tenors
    - Curve Implied Arbitrary Measure Estimates
2. ForwardRateEstimator: ForwardRateEstimator is the interface that exposes the calculation of the Forward Rate for a specific Index. It exposes methods to compute forward rates to a given date/tenor, extract the forward rate index and the Tenor.
3. Turn: Turn implements rate spread at discrete time spans. It contains the turn amount and the start/end turn spans.
4. TurnListDiscountFactor: TurnListDiscountFactor implements the discounting based off of the turns list. Its functions add a turn instance to the current set, and concurrently apply the discount factor inside the range to each relevant turn.
5. RatesLSMM: RatesLSMM contains the Rates specific Latent State MM for the Rates Curve. Current it holds the turn list discount factor.
6. SmoothingCCIS: SmoothingCCIS enhances the Shape Preserving CCIS for smoothing customizations. It exposes the shape preserving discount curve and the smoothing curve stretch parameters.
7. DiscountForwardEstimator: DiscountForwardEstimator exposes the "native" forward curve associated with the specified discount curve. It exposes functionality to extract

forward rate index/tenor, as well as to compute the forward rate implied off of the discount curve.

8. ForwardCurve: ForwardCurve is the stub for the forward curve functionality. It extends the Curve object by exposing the following functions:
    - The name/epoch of the forward rate instance
    - The index/currency/tenor associated with the forward rate instance
    - Forward Rate to a specific date/tenor
    - Generate scenario-tweaked Latent State from the base forward curve corresponding to mode adjusted (flat/parallel/custom) manifest measure/quantification metric.
    - Retrieve array of latent state manifest measure, instrument quantification metric, and the array of calibration components.
    - Set/retrieve curve construction input instrument sets.
9. DiscountCurve: DiscountCurve is the stub for the discount curve functionality. It extends the both the Curve and the DiscountFactorEstimator instances by implementing their functions, and exposing the following:
    - Forward Rate to a specific date/tenor, and effective rate between a date interval
    - Discount Factor to a specific date/tenor, and effective discount factor between a date interval
    - Zero Rate to a specific date/tenor
    - Value Jacobian for Forward rate, discount factor, and zero rate
    - Cross Jacobian between each of Forward rate, discount factor, and zero rate
    - Quote Jacobian to Forward rate, discount factor, and zero rate
    - QM (DF/Zero/Forward) to Quote Jacobian
    - Latent State Quantification Metric, and the quantification metric transformations
    - Implied/embedded ForwardRateEstimator
    - Turns - set/unset/adjust
10. ExplicitBootDiscountCurve: ExplicitBootDiscountCurve exposes the functionality associated with the bootstrapped Discount Curve.
    - Generate a curve shifted using targeted basis at specific nodes

- Generate scenario tweaked Latent State from the base forward curve corresponding to mode adjusted (flat/parallel/custom) manifest measure/quantification metric
- Retrieve array of latent state manifest measure, instrument quantification metric, and the array of calibration components
- Set/retrieve curve construction input instrument sets

11. ZeroCurve: ZeroCurve exposes the node set containing the zero curve node points. In addition to the discount curve functionality that it automatically provides by extension, it provides the functionality to calculate the zero rate.

Bloomberg Sample Package (org.drip.sample.bloomberg)

The Bloomberg Sample Package implements the Bloomberg's calls CDSW, SWPM, and YAS.

1. CDSW: CDSW replicates Bloomberg's CDSW functionality.
2. SWPM: SWPM replicates Bloomberg's SWPM functionality.
3. YAS: YAS replicates Bloomberg's YAS functionality.

Credit Sample Package (org.drip.sample.credit)

The Credit Sample Package demonstrates the core credit analytics functionality – construction of credit curves, pricing of CDS and CDS basket, and retrieve the built-in pre-constructed CDX baskets and CDS closes.

1. CDSBasketAPI: CDSBasketAPI contains a demo of the CDS basket API Sample. It shows the following:
   - Build the IR Curve from the Rates' instruments
   - Build the Component Credit Curve from the CDS instruments
   - Create the basket market parameters and add the named discount curve and the credit curves to it

- Create the CDS basket from the component CDS and their weights
- Construct the Valuation and the Pricing Parameters
- Generate the CDS basket measures from the valuation, the pricer, and the market parameters

2. <u>CDSLiveAndEODAPI</u>: CDSLiveAndEODAPI is a fairly comprehensive sample demonstrating the usage of the EOD and Live CDS Curve API functions. It demonstrates the following:
    - Retrieves all the CDS curves available for the given EOD
    - Retrieves the calibrated credit curve from the CDS instruments for the given CDS curve name, IR curve name, and EOD. Also shows the 10Y survival probability and hazard rate
    - Displays the CDS quotes used to construct the closing credit curve
    - Loads all available credit curves for the given curve ID built from CDS instruments between 2 dates and displays the corresponding 5Y quote
    - Calculate and display the EOD CDS measures for a spot starting CDS based off of a specific credit curve
3. <u>CreditAnalyticsAPI</u>: CreditAnalyticsAPI contains a demo of the CDS Analytics API Sample. It illustrates the following:
    - Credit Curve Creation: From flat Hazard Rate, and from an array of dates and their corresponding survival probabilities
    - Create Credit Curve from CDS instruments, and recover the input measure quotes
    - Create an SNAC CDS, price it, and display the coupon/loss cash flow
4. <u>StandardCDXAPI</u>: StandardCDXAPI contains a demo of the CDS basket API Sample. It shows the following:
    - Construct the CDX.NA.IG 5Y Series 17 index by name and series
    - Construct the on-the-run CDX.NA.IG 5Y Series index
    - List all the built-in CDX - their names and descriptions
    - Construct the on-the run CDX.EM 5Y corresponding to T - 1Y
    - Construct the on-the run ITRAXX.ENERGY 5Y corresponding to T - 7Y
    - Retrieve the full set of date/index series set for ITRAXX.ENERGY

Rates Sample Package (org.drip.sample.rates)

The Rates Sample Package demonstrates the core rates analytics functionality – construction of rates and forward curves (shape preserving/smoothing/transition spline variants) and pricing of rates, treasury, and rates basket products.

1. HaganWestForwardInterpolator: This sample illustrates using the Hagan and West (2006) Estimator. It provides the following functionality:
    - Set up the Predictor ordinates and the response values
    - Construct the rational linear shape control with the specified tension
    - Create the Segment Inelastic design using the Ck and Curvature Penalty Derivatives
    - Build the Array of Segment Custom Builder Control Parameters of the KLK Hyperbolic Tension Basis Type, the tension, the segment inelastic design control, and the shape controller
    - Setup the monotone convex stretch using the above settings, and with no linear inference, no spurious extrema, or no monotone filtering applied
    - Setup the monotone convex stretch using the above settings, and with linear inference, no spurious extrema, or no monotone filtering applied
    - Compute and display the monotone convex output with the linear forward state
    - Compute and display the monotone convex output with the harmonic forward state
2. ShapeDFZeroLocalSmooth: ShapeDFZeroLocalSmooth demonstrates the usage of different local smoothing techniques involved in the discount curve creation. It shows the following:

- Construct the Array of Cash/Swap Instruments and their Quotes from the given set of parameters
- Construct the Cash/Swap Instrument Set Stretch Builder
- Set up the Linear Curve Calibrator using the following parameters:
    - Cubic Exponential Mixture Basis Spline Set

    - $C^k = 2$, Segment Curvature Penalty = 2
    - Quadratic Rational Shape Controller
    - Natural Boundary Setting
- Set up the Akima Local Curve Control parameters as follows:
    - C^1 Akima Monotone Smoothener with spurious extrema elimination and monotone filtering applied
    - Zero Rate Quantification Metric
    - Cubic Polynomial Basis Spline Set
    - $C^k = 2$, Segment Curvature Penalty = 2
    - Quadratic Rational Shape Controller
    - Natural Boundary Setting
- Set up the Harmonic Local Curve Control parameters as follows:
    - C^1 Harmonic Monotone Smoothener with spurious extrema elimination and monotone filtering applied
    - Zero Rate Quantification Metric
    - Cubic Polynomial Basis Spline Set
    - $C^k = 2$, Segment Curvature Penalty = 2
    - Quadratic Rational Shape Controller
    - Natural Boundary Setting
- Set up the Hyman 1983 Local Curve Control parameters as follows:
    - C^1 Hyman 1983 Monotone Smoothener with spurious extrema elimination and monotone filtering applied
    - Zero Rate Quantification Metric
    - Cubic Polynomial Basis Spline Set
    - $C^k = 2$, Segment Curvature Penalty = 2
    - Quadratic Rational Shape Controller
    - Natural Boundary Setting
- Set up the Hyman 1989 Local Curve Control parameters as follows:
    - C^1 Akima Monotone Smoothener with spurious extrema elimination and monotone filtering applied

    - Zero Rate Quantification Metric
    - Cubic Polynomial Basis Spline Set
    - $C^k = 2$, Segment Curvature Penalty = 2
    - Quadratic Rational Shape Controller
    - Natural Boundary Setting
- Set up the Huynh-Le Floch Delimited Local Curve Control parameters as follows:
    - C^1 Huynh-Le Floch Delimited Monotone Smoothener with spurious extrema elimination and monotone filtering applied
    - Zero Rate Quantification Metric
    - Cubic Polynomial Basis Spline Set
    - $C^k = 2$, Segment Curvature Penalty = 2
    - Quadratic Rational Shape Controller
    - Natural Boundary Setting
- Set up the Kruger Local Curve Control parameters as follows:
    - C^1 Kruger Monotone Smoothener with spurious extrema elimination and monotone filtering applied
    - Zero Rate Quantification Metric
    - Cubic Polynomial Basis Spline Set
    - $C^k = 2$, Segment Curvature Penalty = 2
    - Quadratic Rational Shape Controller
    - Natural Boundary Setting
- Construct the Shape Preserving Discount Curve by applying the linear curve calibrator to the array of Cash and Swap Stretches
- Construct the Akima Locally Smoothened Discount Curve by applying the linear curve calibrator and the Local Curve Control parameters to the array of Cash and Swap Stretches and the shape-preserving discount curve
- Construct the Harmonic Locally Smoothened Discount Curve by applying the linear curve calibrator and the Local Curve Control parameters to the array of Cash and Swap Stretches and the shape preserving discount curve

- Construct the Hyman 1983 Locally Smoothened Discount Curve by applying the linear curve calibrator and the Local Curve Control parameters to the array of Cash and Swap Stretches and the shape preserving discount curve
- Construct the Hyman 1989 Locally Smoothened Discount Curve by applying the linear curve calibrator and the Local Curve Control parameters to the array of Cash and Swap Stretches and the shape preserving discount curve
- Construct the Huynh-Le Floch Delimiter Locally Smoothened Discount Curve by applying the linear curve calibrator and the Local Curve Control parameters to the array of Cash and Swap Stretches and the shape preserving discount curve
- Construct the Kruger Locally Smoothened Discount Curve by applying the linear curve calibrator and the Local Curve Control parameters to the array of Cash and Swap Stretches and the shape preserving discount curve
- Cross-Comparison of the Cash/Swap Calibration Instrument "Rate" metric across the different curve construction methodologies
- Cross-Comparison of the Swap Calibration Instrument "Rate" metric across the different curve construction methodologies for a sequence of bespoke swap instruments

3. ShapePreservingDFZeroSmooth: ShapePreservingDFZeroSmooth demonstrates the usage of different shape preserving and smoothing techniques involved in the discount curve creation. It shows the following:
    - Construct the Array of Cash/Swap Instruments and their Quotes from the given set of parameters
    - Construct the Cash/Swap Instrument Set Stretch Builder
    - Set up the Linear Curve Calibrator using the following parameters:
        - Cubic Exponential Mixture Basis Spline Set
        - $C^k = 2$, Segment Curvature Penalty = 2
        - Quadratic Rational Shape Controller
        - Natural Boundary Setting
    - Set up the Global Curve Control parameters as follows:
        - Zero Rate Quantification Metric
        - Cubic Polynomial Basis Spline Set

        - $C^k = 2$, Segment Curvature Penalty = 2
        - Quadratic Rational Shape Controller
        - Natural Boundary Setting
    - Set up the Local Curve Control parameters as follows:
        - C^1 Bessel Monotone Smoothener with no spurious extrema elimination and no monotone filter
        - Zero Rate Quantification Metric
        - Cubic Polynomial Basis Spline Set
        - $C^k = 2$, Segment Curvature Penalty = 2
        - Quadratic Rational Shape Controller
        - Natural Boundary Setting
    - Construct the Shape Preserving Discount Curve by applying the linear curve calibrator to the array of Cash and Swap Stretches
    - Construct the Globally Smoothened Discount Curve by applying the linear curve calibrator and the Global Curve Control parameters to the array of Cash and Swap Stretches and the shape preserving discount curve
    - Construct the Locally Smoothened Discount Curve by applying the linear curve calibrator and the Local Curve Control parameters to the array of Cash and Swap Stretches and the shape preserving discount curve
    - Cross-Comparison of the Cash/Swap Calibration Instrument "Rate" metric across the different curve construction methodologies
    - Cross-Comparison of the Swap Calibration Instrument "Rate" metric across the different curve construction methodologies for a sequence of bespoke swap instruments
4. <u>CustomDiscountCurveBuilder</u>: CustomDiscountCurveBuilder discount curve calibration and input instrument calibration quote recovery. It shows the following:
    - Construct the Array of Cash/Swap Instruments and their Quotes from the given set of parameters
    - Construct the Cash/Swap Instrument Set Stretch Builder
    - Set up the Linear Curve Calibrator using the following parameters:
        - Cubic Exponential Mixture Basis Spline Set

        - $C^k = 2$, Segment Curvature Penalty = 2
        - Quadratic Rational Shape Controller
        - Natural Boundary Setting
    - Construct the Shape Preserving Discount Curve by applying the linear curve calibrator to the array of Cash and Swap Stretches
    - Cross-Comparison of the Cash/Swap Calibration Instrument "Rate" metric across the different curve construction methodologies
5. <u>CustomDiscountCurveReconciler</u>: CustomDiscountCurveReconciler demonstrates the multi-stretch transition custom discount curve construction, turns application, discount factor extraction, and calibration quote recovery. It shows the following steps:
    - Setup the linear curve calibrator
    - Setup the cash instruments and their quotes for calibration
    - Setup the cash instruments stretch latent state representation - this uses the discount factor quantification metric and the "rate" manifest measure
    - Setup the swap instruments and their quotes for calibration
    - Setup the swap instruments stretch latent state representation - this uses the discount factor quantification metric and the "rate" manifest measure
    - Calibrate over the instrument set to generate a new overlapping latent state span instance
    - Retrieve the "cash" stretch from the span
    - Retrieve the "swap" stretch from the span
    - Create a discount curve instance by converting the overlapping stretch to an exclusive non-overlapping stretch
    - Compare the discount factors and their monotonicity emitted from the discount curve, the non-overlapping span, and the "swap" stretch across the range of tenor predictor ordinates
    - Cross-Recovery of the Cash Calibration Instrument "Rate" metric across the different curve construction methodologies
    - Cross-Recovery of the Swap Calibration Instrument "Rate" metric across the different curve construction methodologies

- Create a turn list instance and add new turn instances
- Update the discount curve with the turn list
- Compare the discount factor implied the discount curve with and without applying the turns adjustment

6. <u>DiscountCurveQuoteSensitivity</u>: DiscountCurveQuoteSensitivity demonstrates the calculation of the discount curve sensitivity to the calibration instrument quotes. It does the following:
   - Construct the Array of Cash/Swap Instruments and their Quotes from the given set of parameters
   - Construct the Cash/Swap Instrument Set Stretch Builder.
   - Set up the Linear Curve Calibrator using the following parameters:
     - Cubic Exponential Mixture Basis Spline Set
     - $C^k = 2$, Segment Curvature Penalty = 2
     - Quadratic Rational Shape Controller
     - Natural Boundary Setting
   - Construct the Shape Preserving Discount Curve by applying the linear curve calibrator to the array of Cash and Swap Stretches
   - Cross-Comparison of the Cash/Swap Calibration Instrument "Rate" metric across the different curve construction methodologies
   - Display of the Cash Instrument Discount Factor Quote Jacobian Sensitivities
   - Display of the Swap Instrument Discount Factor Quote Jacobian Sensitivities
7. <u>TemplatedDiscountCurveBuilder</u>: TemplatedDiscountCurveBuilder sample demonstrates the usage of the different pre-built Discount Curve Builders. It shows the following:
   - Construct the Array of Cash Instruments and their Quotes from the given set of parameters
   - Construct the Array of Swap Instruments and their Quotes from the given set of parameters
   - Construct the Cubic Tension KLK Hyperbolic Discount Factor Shape Preserver

    - Construct the Cubic Tension KLK Hyperbolic Discount Factor Shape Preserver with Zero Rate Smoothening applied
    - Construct the Cubic Polynomial Discount Factor Shape Preserver
    - Construct the Cubic Polynomial Discount Factor Shape Preserver with Zero Rate Smoothening applied
    - Construct the Discount Curve using the Bear Sterns' DENSE Methodology
    - Construct the Discount Curve using the Bear Sterns' DUALDENSE Methodology
    - Cross-Comparison of the Cash Calibration Instrument "Rate" metric across the different curve construction methodologies
    - Cross-Comparison of the Swap Calibration Instrument "Rate" metric across the different curve construction methodologies
    - Cross-Comparison of the generated Discount Factor across the different curve construction Methodologies for different node points

8. <u>CustomForwardCurveBuilder</u>: CustomForwardCurveBuilder contains the sample demonstrating the full functionality behind creating highly customized spline based forward curves.

   The first sample illustrates the creation and usage of the xM-6M Tenor Basis Swap:

   - Construct the 6M-xM float-float basis swap
   - Calculate the corresponding starting forward rate off of the discount curve
   - Construct the shape preserving forward curve off of Cubic Polynomial Basis Spline
   - Construct the shape preserving forward curve off of Quartic Polynomial Basis Spline
   - Construct the shape preserving forward curve off of Hyperbolic Tension Based Basis Spline
   - Set the discount curve based component market parameters
   - Set the discount curve + cubic polynomial forward curve based component market parameters
   - Set the discount curve + quartic polynomial forward curve based component market parameters

- Set the discount curve + hyperbolic tension forward curve based component market parameters
- Compute the following forward curve metrics for each of cubic polynomial forward, quartic polynomial forward, and KLK Hyperbolic tension forward curves:
    - Reference Basis Par Spread
    - Derived Basis Par Spread
- Compare these with a) the forward rate off of the discount curve, b) The LIBOR rate, and c) The Input Basis Swap Quote

The second sample illustrates how to build and test the forward curves across various tenor basis. It shows the following steps:

- Construct the Discount Curve using its instruments and quotes
- Build and run the sampling for the 1M-6M Tenor Basis Swap from its instruments and quotes
- Build and run the sampling for the 3M-6M Tenor Basis Swap from its instruments and quotes
- Build and run the sampling for the 6M-6M Tenor Basis Swap from its instruments and quotes
- Build and run the sampling for the 12M-6M Tenor Basis Swap from its instruments and quotes

9. RatesAnalyticsAPI: RatesAnalyticsAPI contains a demo of the Rates Analytics API Usage. It shows the following:
    - Build a discount curve using: cash instruments only, EDF instruments only, IRS instruments only, or all of them strung together
    - Re-calculate the component input measure quotes from the calibrated discount curve object
    - Compute the PVDF Wengert Jacobian across all the instruments used in the curve construction
10. TreasuryCurveAPI: TreasuryCurveAPI contains a demo of construction and usage of the treasury discount curve from government bond inputs. It shows the following:
    - Create on-the-run TSY bond set

- Calibrate a discount curve off of the on-the-run yields and calculate the implied zeroes and DF's
- Price an off-the-run TSY

11. RatesLiveAndEODAPI: RatesLiveAndEODAPI contains the sample API demonstrating the usage of the Rates Live and EOD functions. It does the following:
    - Pulls all the closing rates curve names (of any type, incl. TSY) that exist for a given date
    - Load the full IR curve created from all the single currency rate quotes (except TSY) for the given currency and date
    - Calculate the discount factor to an arbitrary date using the constructed curve
    - Retrieve the components and their quotes that went into constructing the curve, and display them
    - Load all the rates curves available between the dates for the currency specified, and step through
    - Load all the Cash quotes available between the dates for the currency specified, and step through
    - Load all the EDF quotes available between the dates for the currency specified, and step through
    - Load all the IRS quotes available between the dates for the currency specified, and step through
    - Load all the TSY quotes available between the dates for the currency specified, and step through
12. MultiLegSwapAPI: MultiLegSwapAPI illustrates the creation, invocation, and usage of the MultiLegSwap. It shows how to:
    - Create the Discount Curve from the rates instruments
    - Set up the valuation and the market parameters
    - Create the Rates Basket from the fixed/float streams
    - Value the Rates Basket

Curve Regression Package (org.drip.regression.curve)

The Curve Regression Package demonstrates the core curve regression functionality – regression of discount curve, credit curve, FX forward/basis curve, and zero curves.

1. <u>DiscountCurveRegressor</u>: DiscountCurveRegressor implements the regression set analysis for the Discount Curve. DiscountCurveRegressor regresses 11 scenarios:
   - #1: Create the discount curve from a set 30 instruments (cash/future/swap)
   - #2: Create the discount curve from a flat discount rate
   - #3: Create the discount curve from a set of discount factors
   - #4: Create the discount curve from the implied discount rates
   - #5: Extract the discount curve instruments and quotes
   - #6: Create a parallel shifted discount curve
   - #7: Create a rate shifted discount curve
   - #8: Create a basis rate shifted discount curve
   - #9: Create a node tweaked discount curve
   - #10: Compute the effective discount factor between 2 dates
   - #11: Compute the effective implied rate between 2 dates
2. <u>ZeroCurveRegressor</u>: ZeroCurveRegressor implements the regression analysis set for the Zero Curve. The regression tests consists of the following:
   - Build a discount curve, followed by the zero curve
   - Regressor #1: Compute zero curve discount factors
   - Regressor #2: Compute zero curve zero rates
3. <u>CreditCurveRegressor</u>: CreditCurveRegressor implements the regression set analysis for the Credit Curve. CreditCurveRegressor regresses 12 scenarios:
   - #1: Create an SNAC CDS
   - #2: Create the credit curve from a set of CDS instruments
   - #3: Create the credit curve from a flat hazard rate
   - #4: Create the credit curve from a set of survival probabilities
   - #5: Create the credit curve from an array of hazard rates
   - #6: Extract the credit curve instruments and quotes

- #7: Create a parallel hazard shifted credit curve
- #8: Create a parallel quote shifted credit curve
- #9: Create a node tweaked credit curve
- #10: Set a specific default date on the credit curve
- #11: Compute the effective survival probability between 2 dates
- #12: Compute the effective hazard rate between 2 dates

4. FXCurveRegressor: FXCurveRegressor implements the regression analysis set for the FX Curve. FXCurveRegressor implements 3 regression tests:
   - #1: FX Basis and FX Curve Creation: Construct a FX forward Curve from an array of FX forward nodes and the spot
   - #2: Imply the FX Forward given the domestic and foreign discount curves
   - #3a: Compute the domestic and foreign basis given the market FX forward
   - #3b: Build the domestic/foreign basis curve given the corresponding basis nodes
   - #3c: Imply the array of FX forward points/PIPs from the array of basis and domestic/foreign discount curves
5. CreditAnalyticsRegressionEngine: CreditAnalyticsRegressionEngine implements the RegressionEngine for the curve regression. It adds the CreditCurveRegressor, DiscountCurveRegressor, FXCurveRegressor, and ZeroCurveRegressor, and launches the regression engine.

Product Curve Jacobian Regression Package (org.drip.regression.curveJacobian)

The Product Curve Jacobian Regression package carries out regression across the core suite of products Jacobian to the curve– Cash, EDF, and Fix-float IRS. It also implements the Curve Jacobian Regression Engine.

1. CashJacobianRegressorSet: CashJacobianRegressorSet implements the regression analysis set for the Cash product related Sensitivity Jacobians. Specifically, it computes the PVDF micro-Jack.

2. EDFJacobianRegressorSet: EDFJacobianRegressorSet implements the regression analysis set for the EDF product related Sensitivity Jacobians. Specifically, it computes the PVDF micro-Jack.
3. IRSJacobianRegressorSet: IRSJacobianRegressorSet implements the regression analysis set for the IRS product related Sensitivity Jacobians. Specifically, it computes the PVDF micro-Jack.
4. DiscountCurveJacobianRegressorSet: DiscountCurveJacobianRegressorSet implements the regression analysis for the full discount curve (built from cash/future/swap) Sensitivity Jacobians. Specifically, it computes the PVDF micro-Jack.
5. CurveJacobianRegressionEngine: CurveJacobianRegressionEngine implements the RegressionEngine for the curve Jacobian regression. It adds the CashJacobianRegressorSet, the EDFJacobianRegressorSet, the IRSJacobianRegressorSet, and the DiscountCurveJacobianRegressorSet, and launches the regression engine.