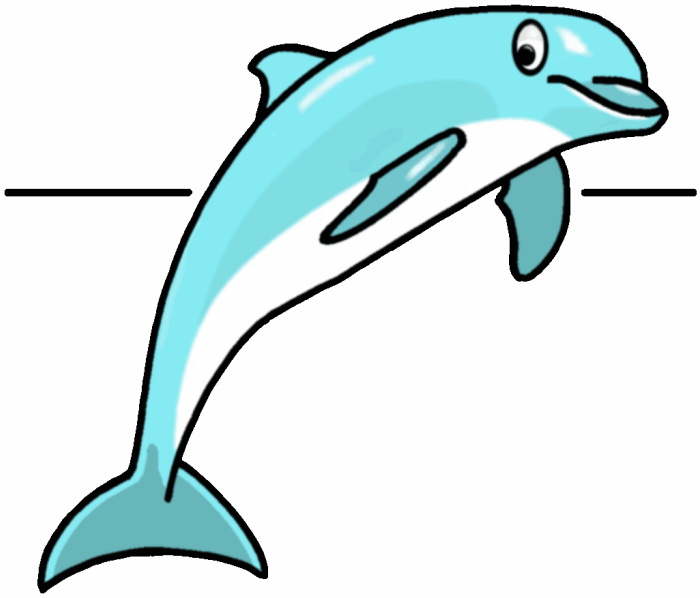
CYCLONE
Fluid Dynamics

Dolfyn Test Results

DRAFT



CFD-11xxxx
21st January 2011

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Dolfyn Test Results

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Cyclone Fluid Dynamics BV

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H.W. Krüs

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1 Verification with equidistant Lid Driven Cavity tests

Following Roache[1], an error analysis has been performed using 'Richardson Extrapolation' on various meshes and convective differencing schemes. All the meshes are simple orthogonal and equidistant. Although dolfyn contains non-orthogonal corrections etcetera, still this is a valuable test because all additions should not break the basic fundament of the code. The reference data is by Ghia, Ghia, and Shin[2].

The cavity has dimensions of one by one and the lid has a velocity of one, leaving the (laminar) viscosity of the fluid with density one to determine the Reynolds number ($Re = \rho V_{\text{lid}} L_{\text{lid}}/\mu$). The calculations were done with Gauss for the gradients and various convective differencing schemes:

UD Classic standard first order Upwind Differencing.

CD 0.8 Blend of 80% second order Central Differencing and 20% first order upwind differencing.

CD1 Pure second order Central Differencing.

LUD Second order Linear Upwind Differencing with a Convection Bounded Criterion (UD outside the Normalised Variable range of $0 \leq \tilde{\Phi}_C \leq 1$).

Gamma The Gamma CBC differencing scheme (blending with CD).

MinMod The MinMod CBC differencing scheme (based on LUD and CD).

LUX Pure LUD not based on NVD..

Shown are the u component velocity profiles half way the cavity ($x = 0.5$, $0 \leq y \leq 1$) for all the schemes at the finest mesh (128x128) and UD and LUX only as a function of the mesh (8x8, 16x16, 32x32, 64x64, 128x128). And the development of the interpolated u component at $x = 0.5, y = 0.5$. The latter value has been linearly interpolated (which might have some effects on the final results) and is shown as a function of mesh size h (linear) and h^2 (quadratic). A second order method will have to show up as a straight line at the smallest meshes.

[1] P.J. Roache, *Verification and Validation in Computational Science and Engineering*, Hermosa Publishers, Albuquerque NM, 1998

[2] U. Ghia, K.N. Ghia, C.T. Shin, *High-Re solutions for incompressible flow using the Navier-Stokes equations and a multigrid method*, Journal of Computational Physics, 1982

Also shown are streamlines based on the LUX data like in Figure 1.1. They were made with OpenDX after the cell-centered velocities have been interpolated to the nodes and creating streamlines (the 'post' and 'streamline' modules of OpenDX). Because of this procedure the streamlines will lose some of their accuracy near the walls. However nice closed streamlines do show up in the center of the main vortex.

In all this study is the result of at least 432 runs.

Brief discussion of the results:

Re 25 A very very viscous flow. The flow is dominated by the (second order) viscous forces. All convective schemes coincide on the largest mesh (128x128). The first order behaviour of UD is clearly visible as well as the second order nature of CD1 and LUX (note that CD 0.8 lays nicely in between of CD1 and UD).

Re 100 The first case were data is available from Ghia et.al. As can be seen LUX on at a mesh of 32x32, or even 16x16, already produces the final result. The Richardson Extrapolation curves clearly supports this. Note the start of the two lower corner vortices.

Re 400 A 16 times lower viscosity compared to Re 25 starts to show some differences especially with UD; a good result is only possible at the finest 128x128 mesh. The second order schemes CD1 and LUX reproduce the reference data exactly, closely followed by CD 0.8. The LUX results are already there on the 64x64 mesh (but the 32x32 are not bad either). The lower right corner vortex increases.

Re 1,000 Basically the same results as for Re 400. In the top left the vortex is about to appear.

Re 3,200 Now the differences between the schemes get very clear. Note that the NVD blended schemes follow the UD scheme whereas the two unbounded second order schemes provide the best result (again followed by the CD/UD blend). The LUX scheme is the best and the Richardson Extrapolation curve of h^2 shows a 'tail' for the CD1 scheme. The latter is now starting to deteriorate.

Re 5,000 Now only LUX is the only one which follows the data, and only on the finest 128x128 mesh. The 'tail' of CD1 in the h^2 Richardson Extrapolation curve is more pronounced. In the paper by Ghia et.al the results are show on a 257x257 mesh and a second vortex starts to appear in the lower left corner (see Figure 1.3).

Re 7,500 At Reynolds 7,500 and on a mesh of 128x128 even LUX is not able to reproduce the reference data. Also instabilities start to occur; see Figure 1.10.

Re 10,000 The results of Reynolds 10,000 resemble the Reynolds 7,500 results. The results in Figure 1.11 are based on a 256x256 mesh and Figure 1.12 shows the corresponding streamlines. It is clearly visible that the upwind

and CBC upwinded schemes are the most stable. The (unboudded) second order schemes CD1 and LUX show unsteady effects; the latter is also visisble in the residual drop which do not reach machine accuracy levels anymore. The results in the reference paper are based on a high order upwind scheme with a larger stencil (with a face based unstructured solver one has to restrict to a small stencil). Nevertehless at some point instabilities as can be seen in Figure 1.12 will have to pop up at some point.

General conlusion is that dolfyn is a second order accurate code which provides for this particular case and mesh topology very accurate and correct results. Also for testing purposes the lid driven cavity can be used at medium Reynolds numbers; for example only at Re 400 or the combination Re 100 and Re 1,000.

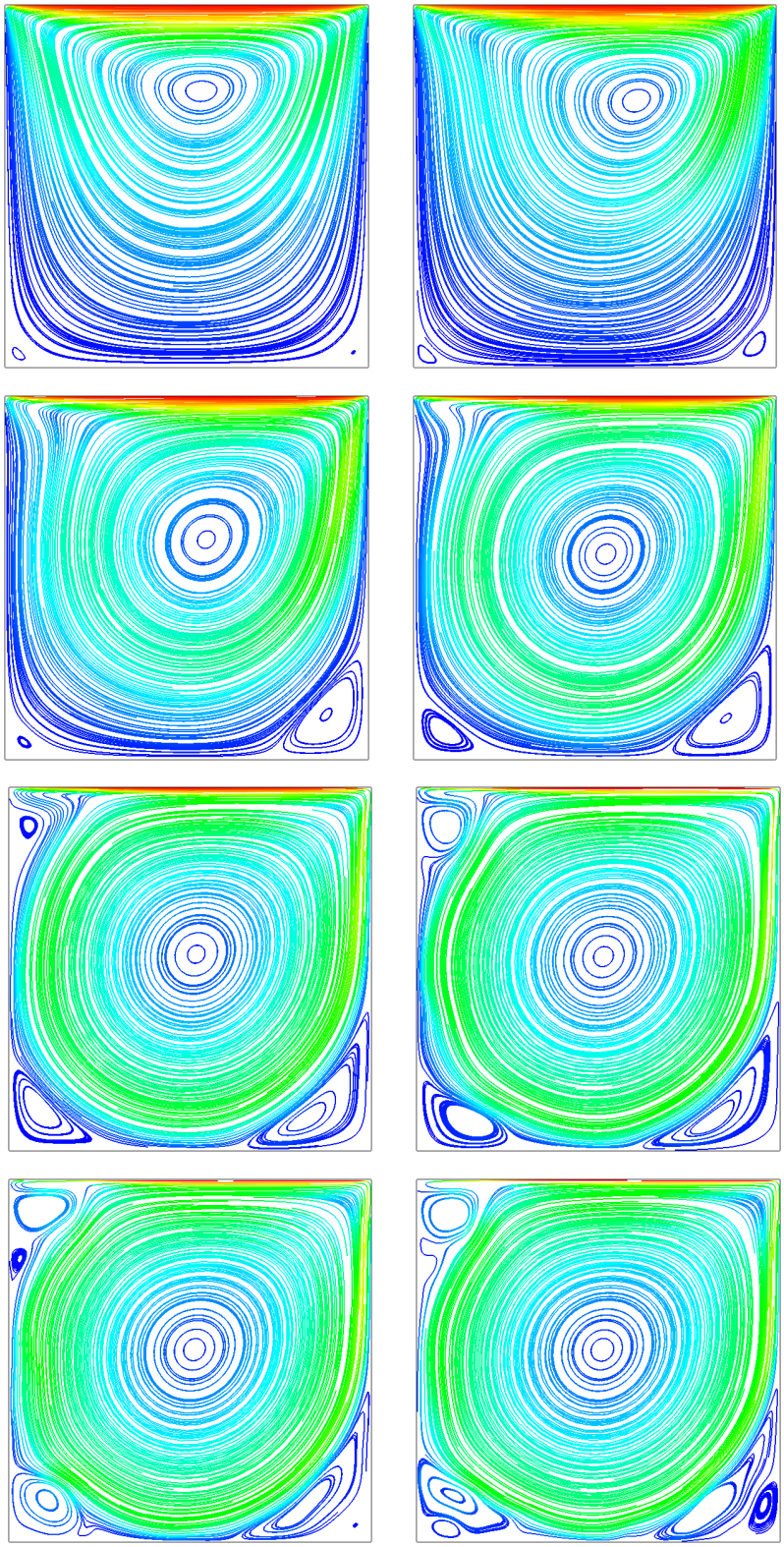


Figure 1.1: Streamlines at Reynolds 25, 100, 400, 1,000, 3,200, 5,000, 7,500 and 10,000 with 128x128 and LUX

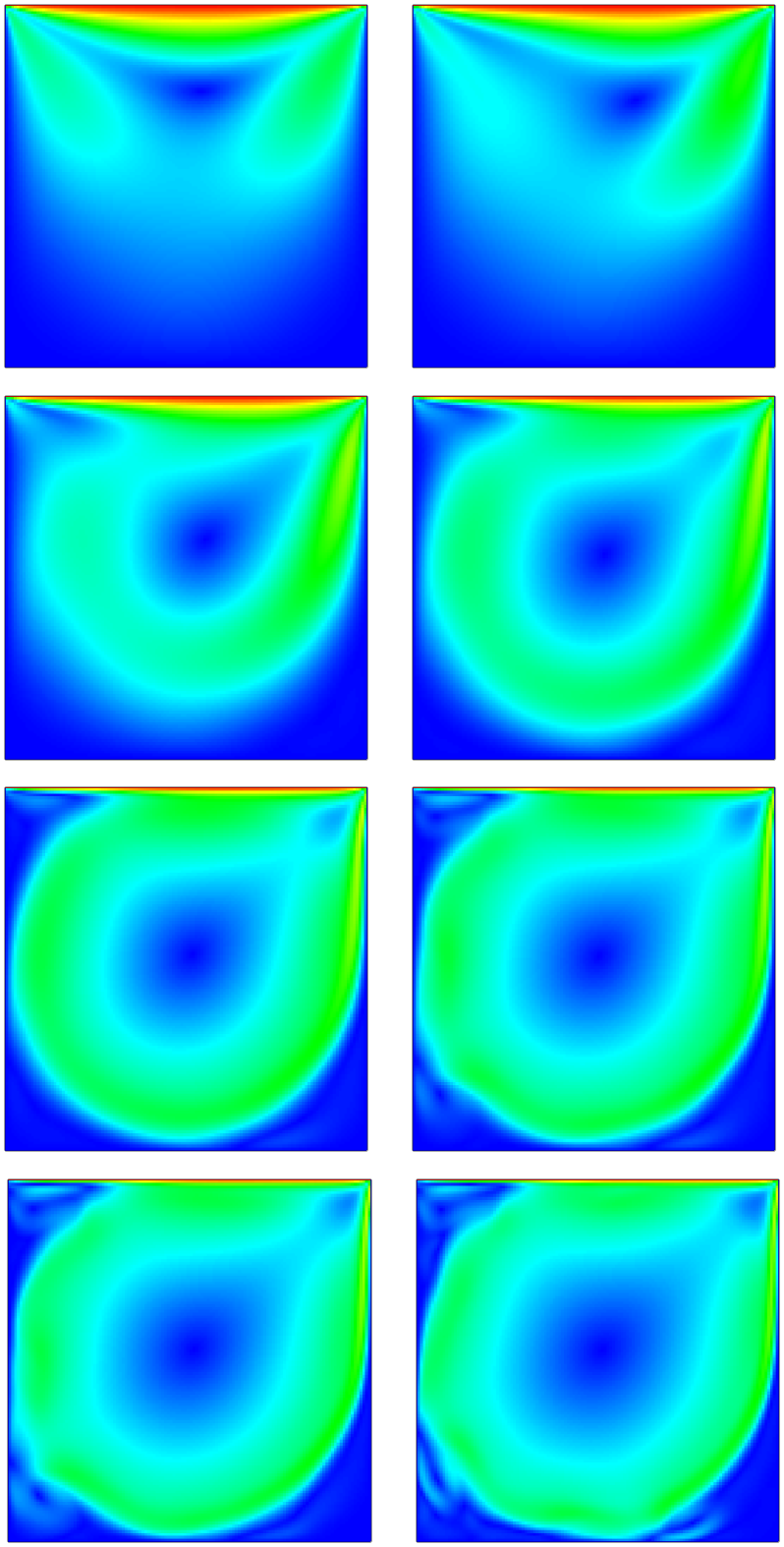


Figure 1.2: Velocity magnitude at Reynolds 25, 100, 400, 1,000, 3,200, 5,000, 7,500 and 10,000 with 128x128 and LUX

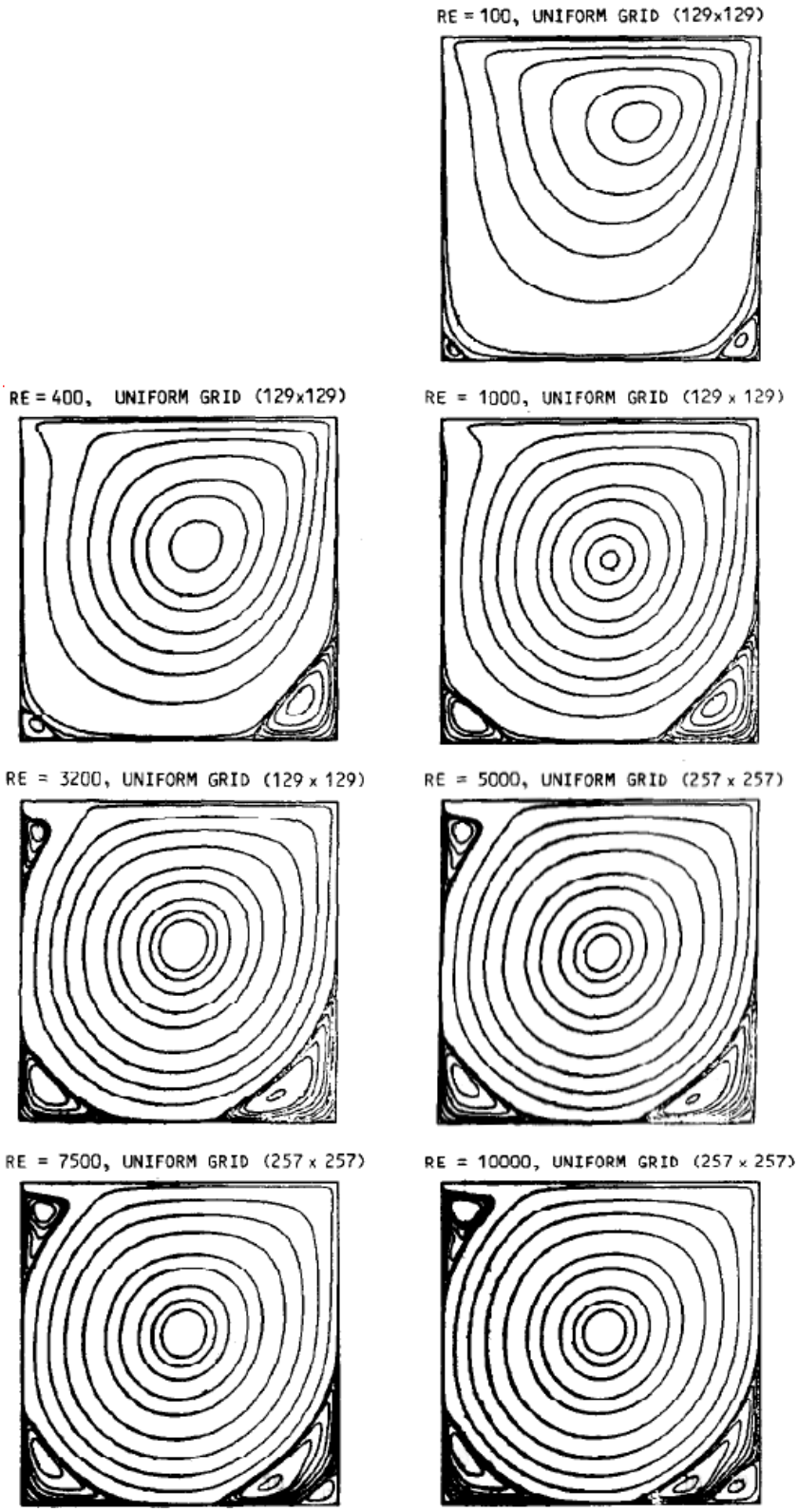


Figure 1.3: Results from Ghia et.al

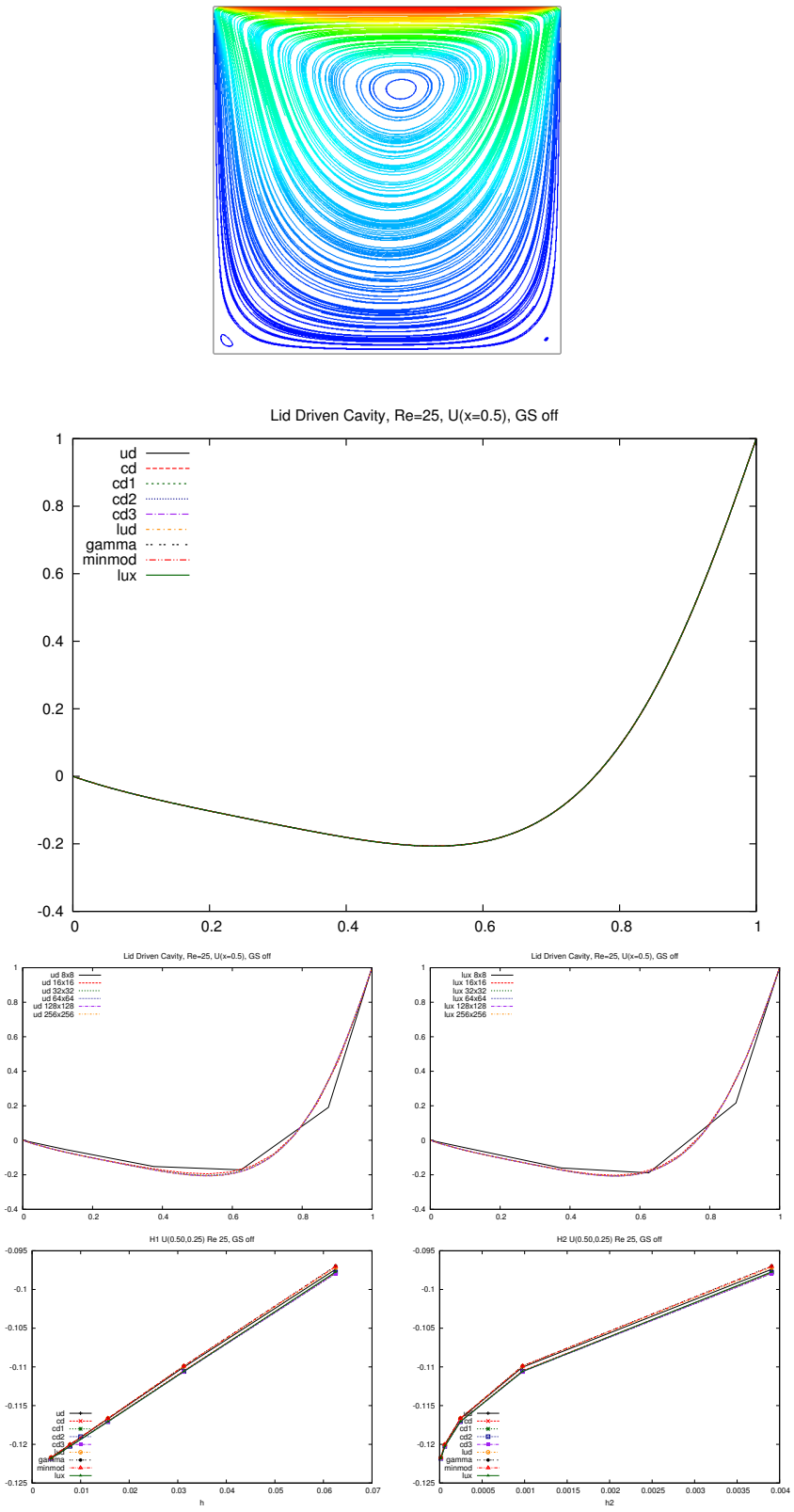


Figure 1.4: Reynolds 25

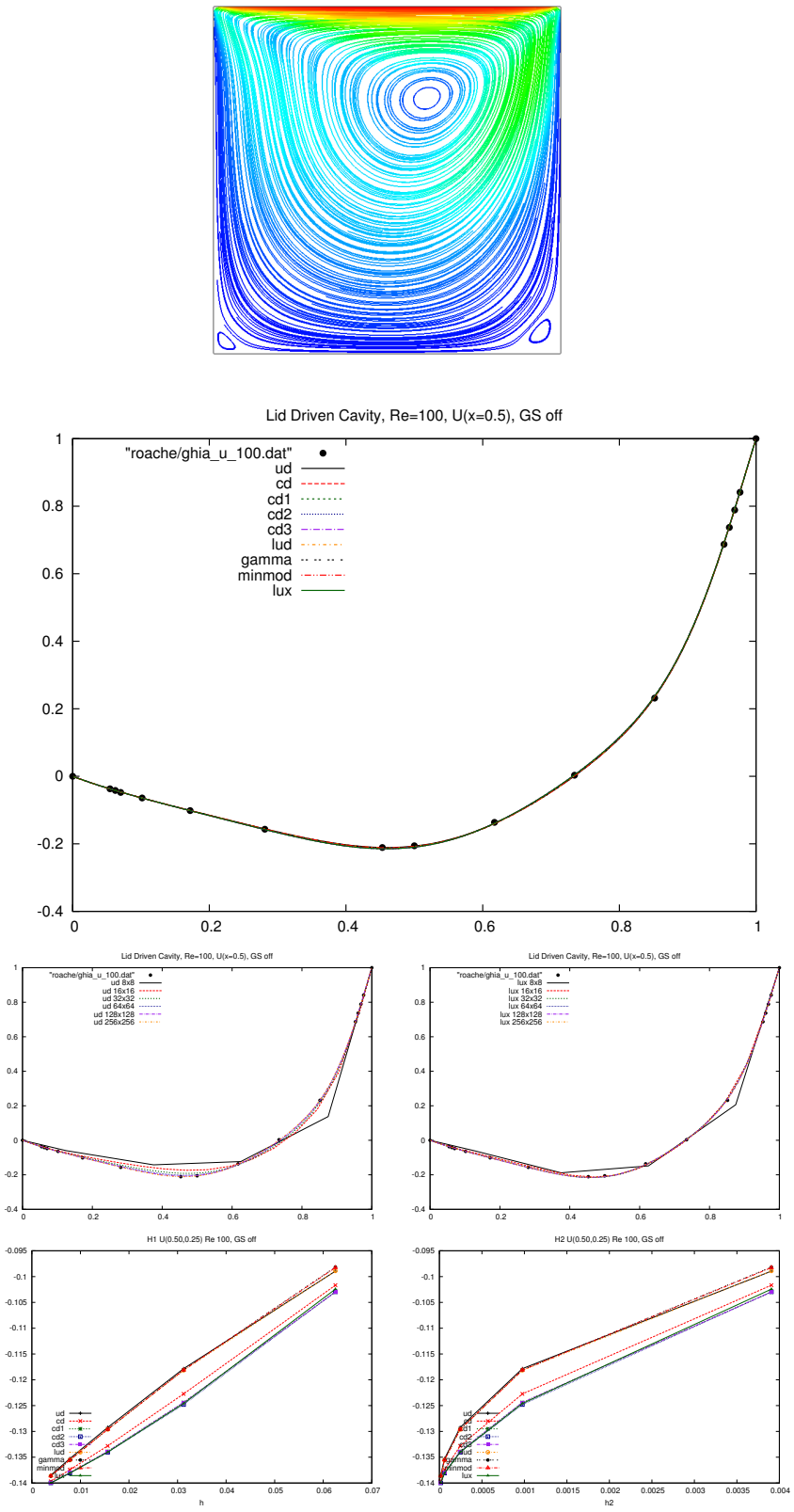


Figure 1.5: Reynolds 100

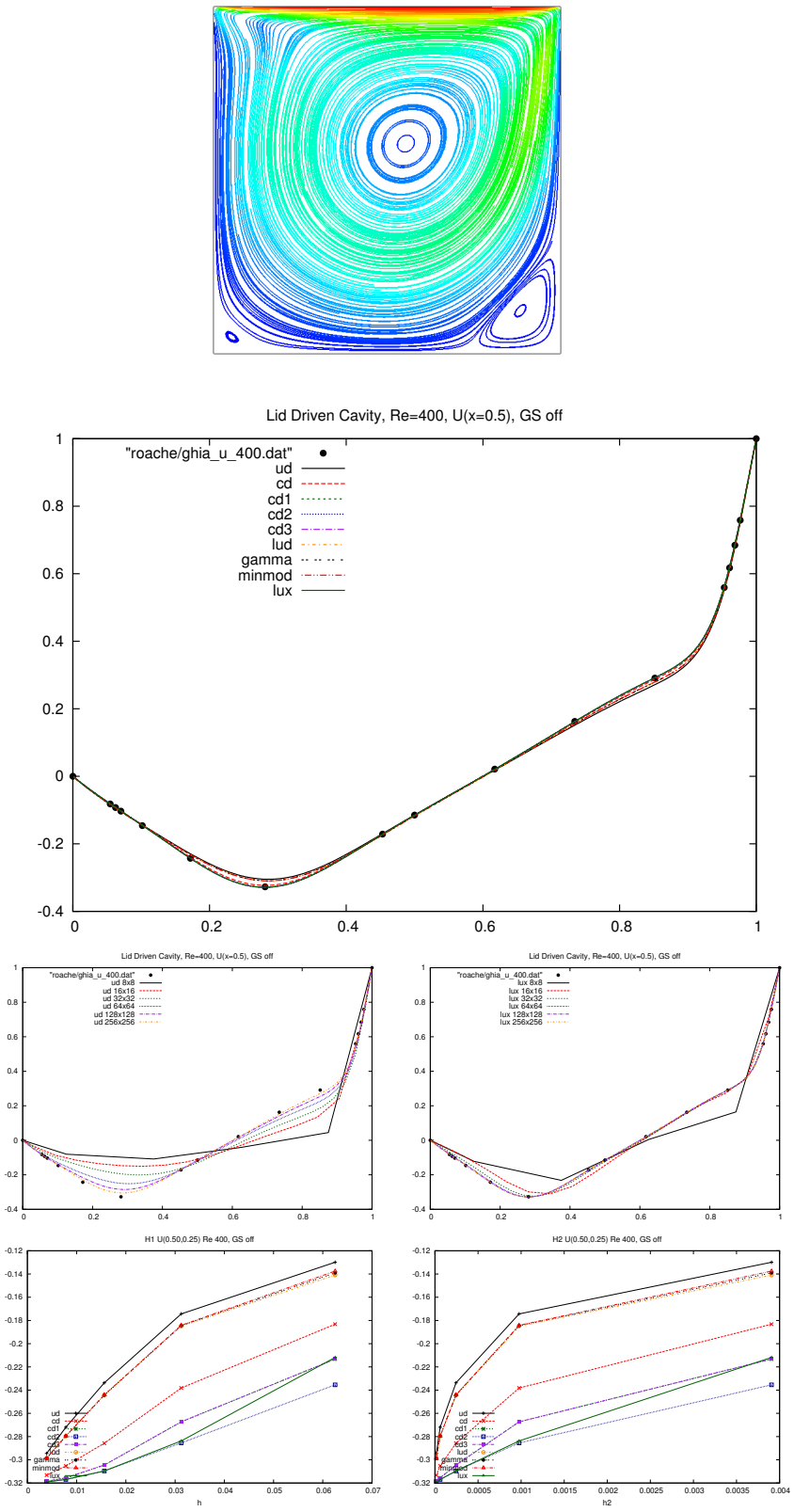


Figure 1.6: Reynolds 400

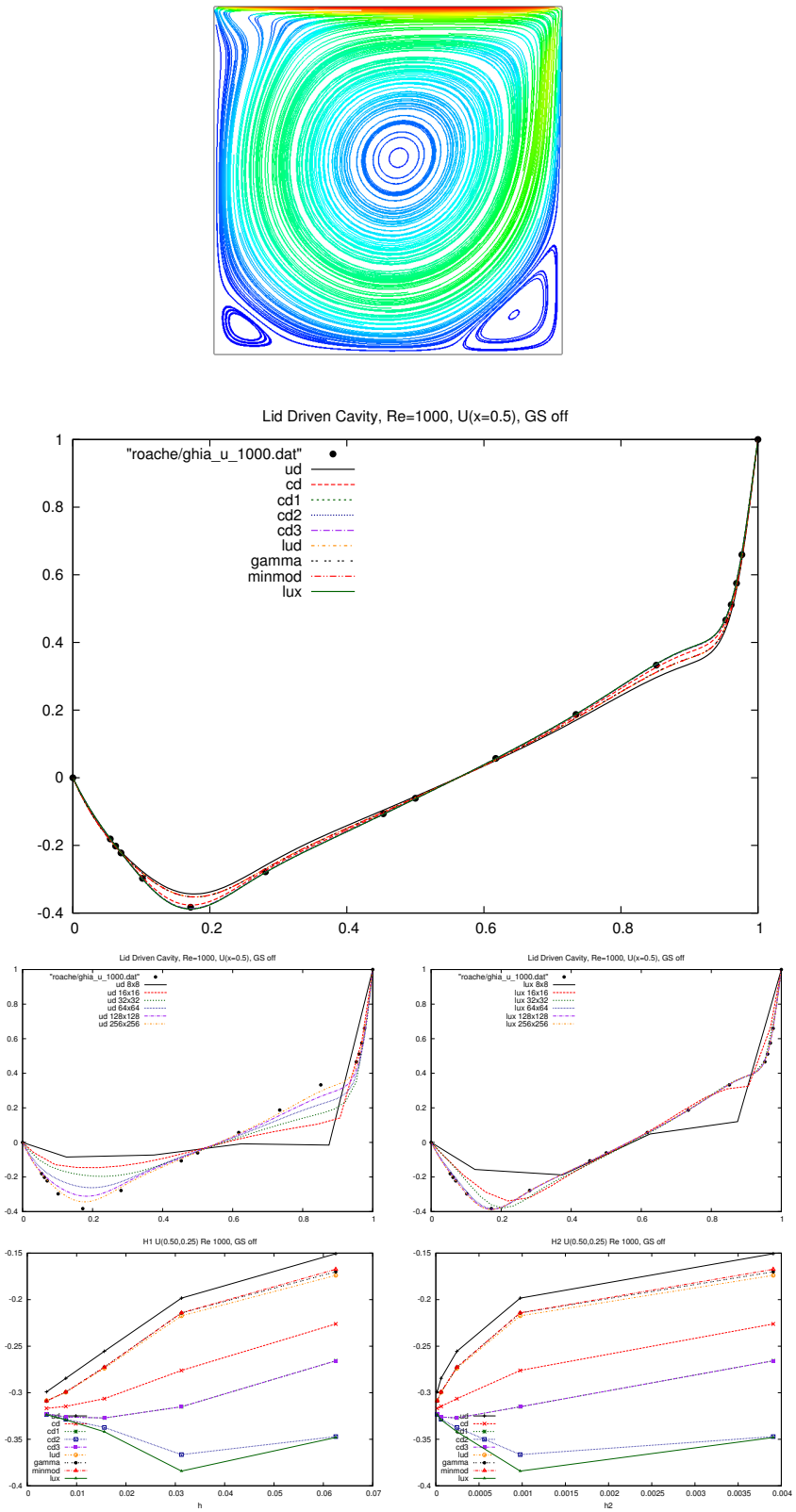
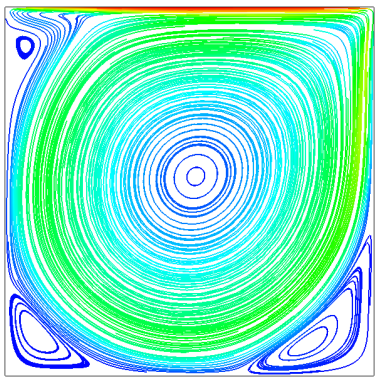


Figure 1.7: Reynolds 1,000



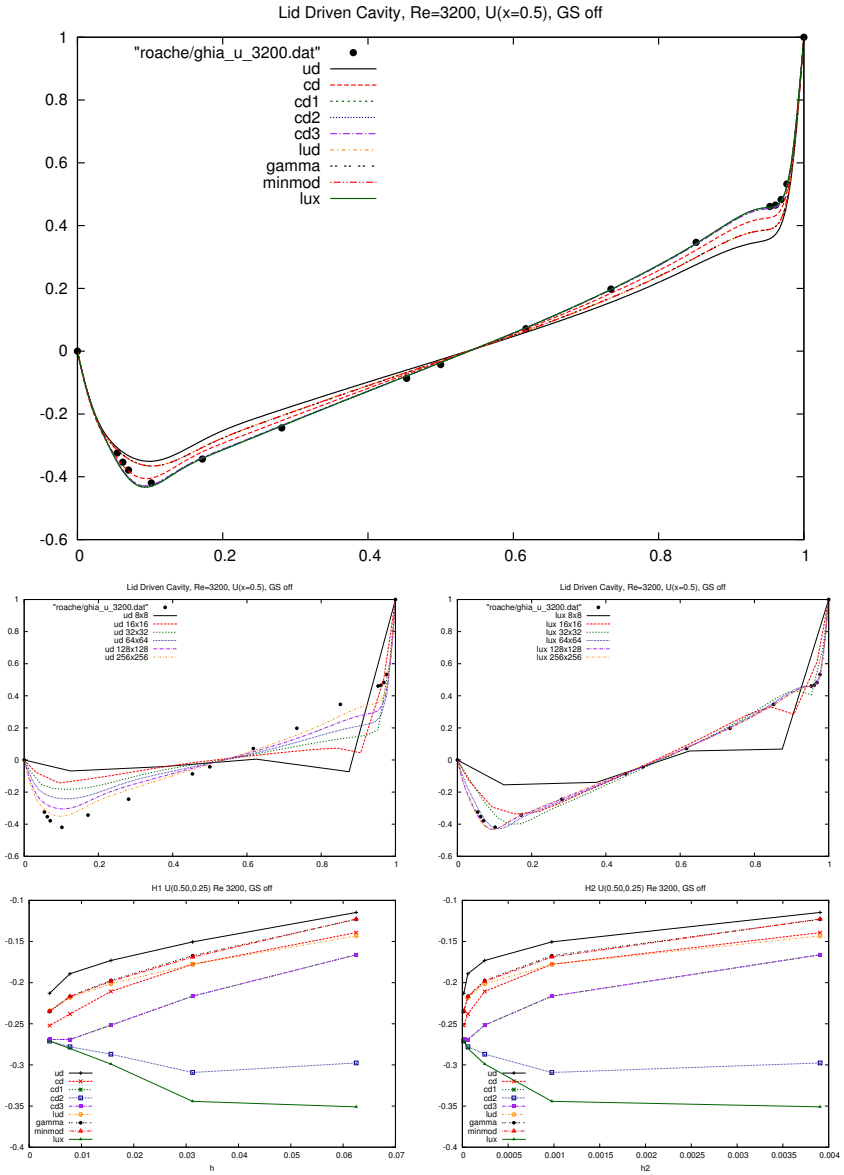


Figure 1.8: Reynolds 3,200

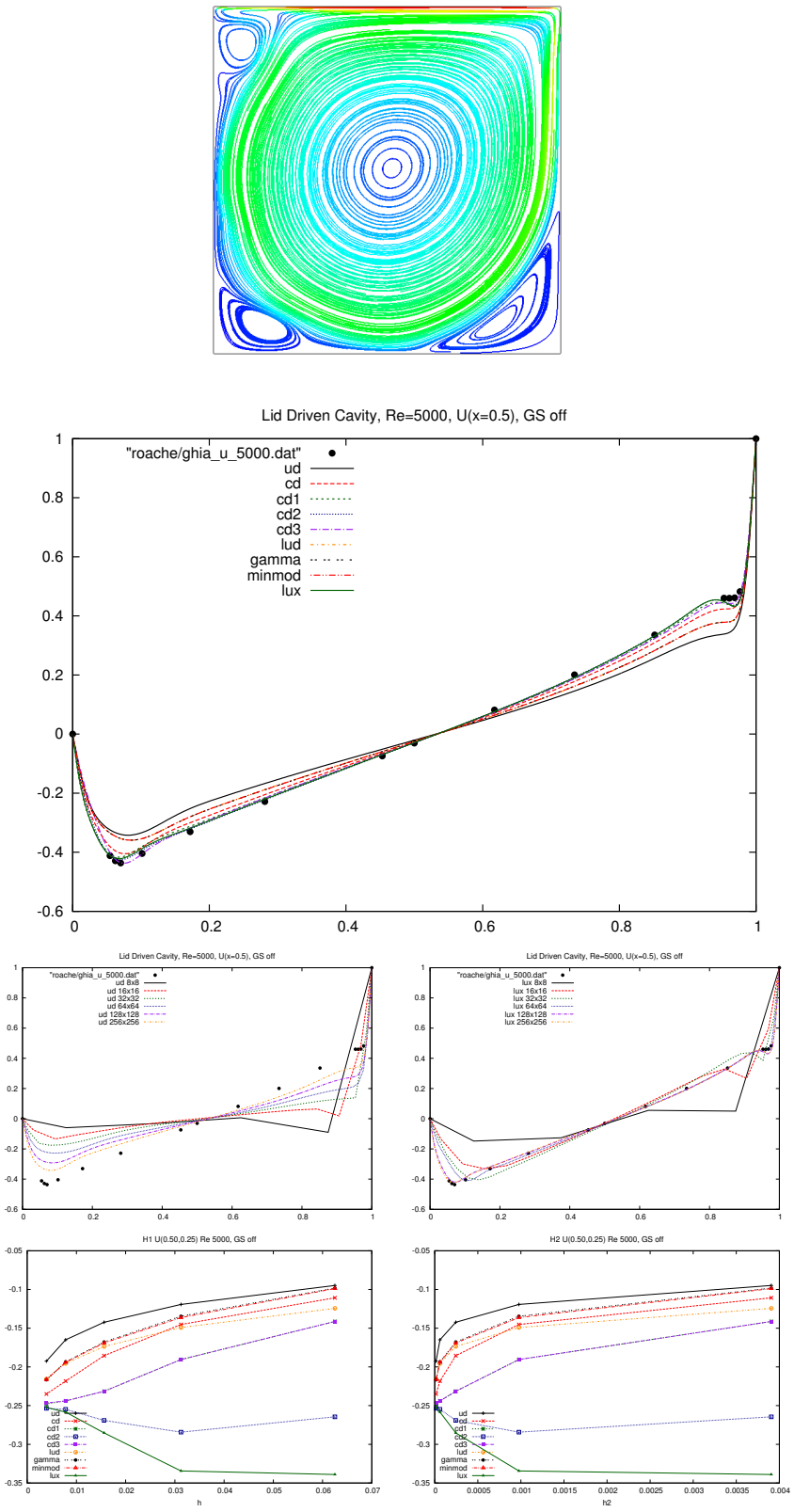
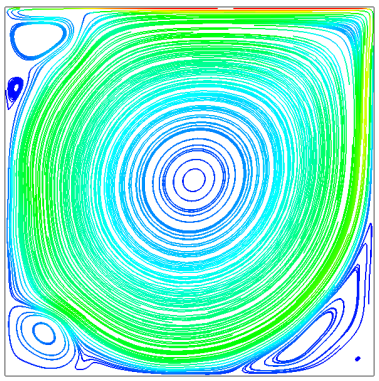


Figure 1.9: Reynolds 5,000



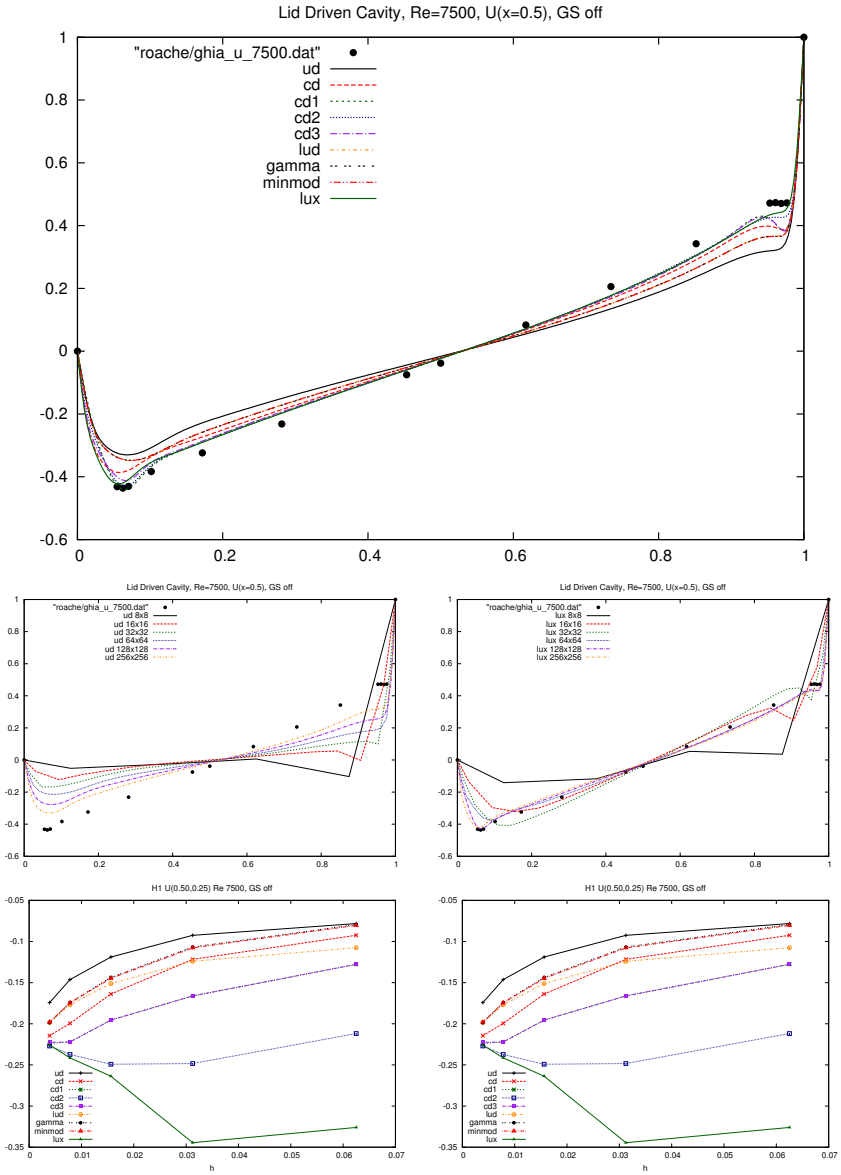


Figure 1.10: Reynolds 7,500

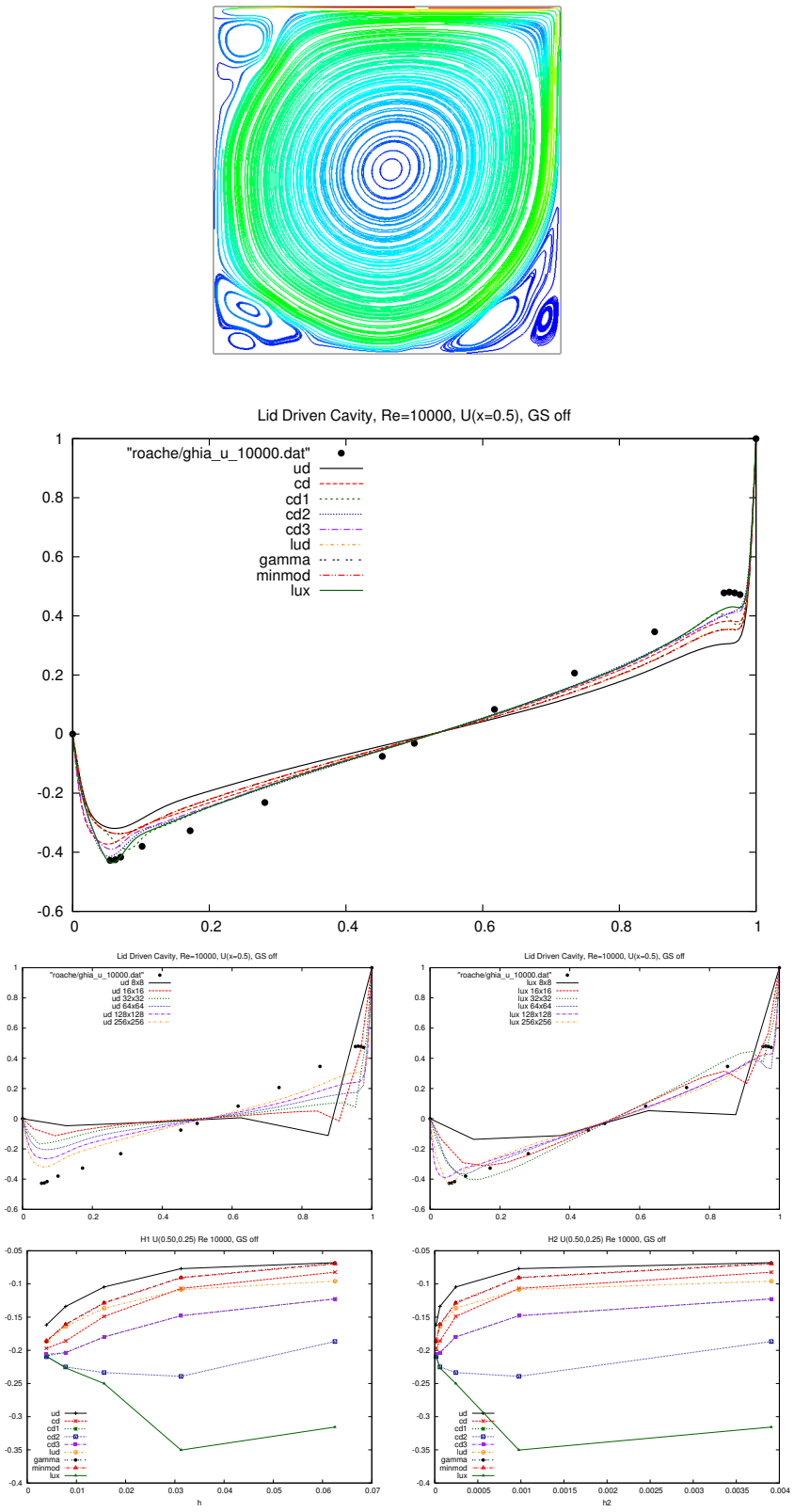


Figure 1.11: Reynolds 10,000

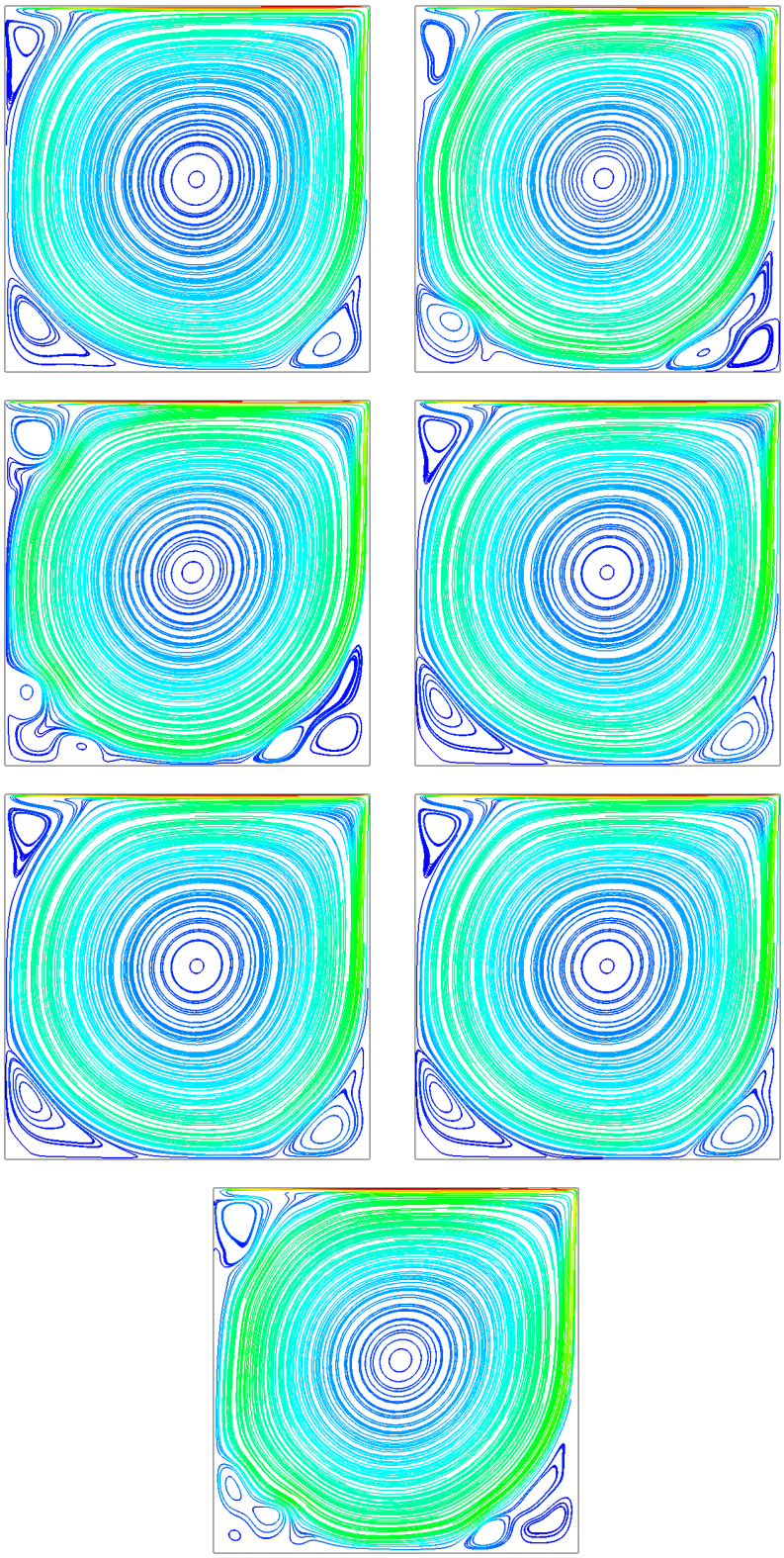


Figure 1.12: Reynolds 10,000 at 256x256 and influence of differencing scheme (UD, CD, CD1, LUD, Gamma, MinMod, LUX)

2 C1 Unstructured

Flow from left to right with scalar temperature convection top 21 C and bottom half 20 C. Uniform unity velocity. In the middle the mesh is tilted by 45 degrees, beginning just after the splitter plate at distance 1. Old test which shows the peculiar upstream influence of CD type and based schemes.

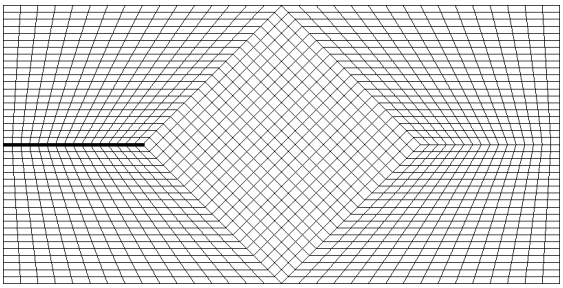


Figure 2.1: Mesh model c1

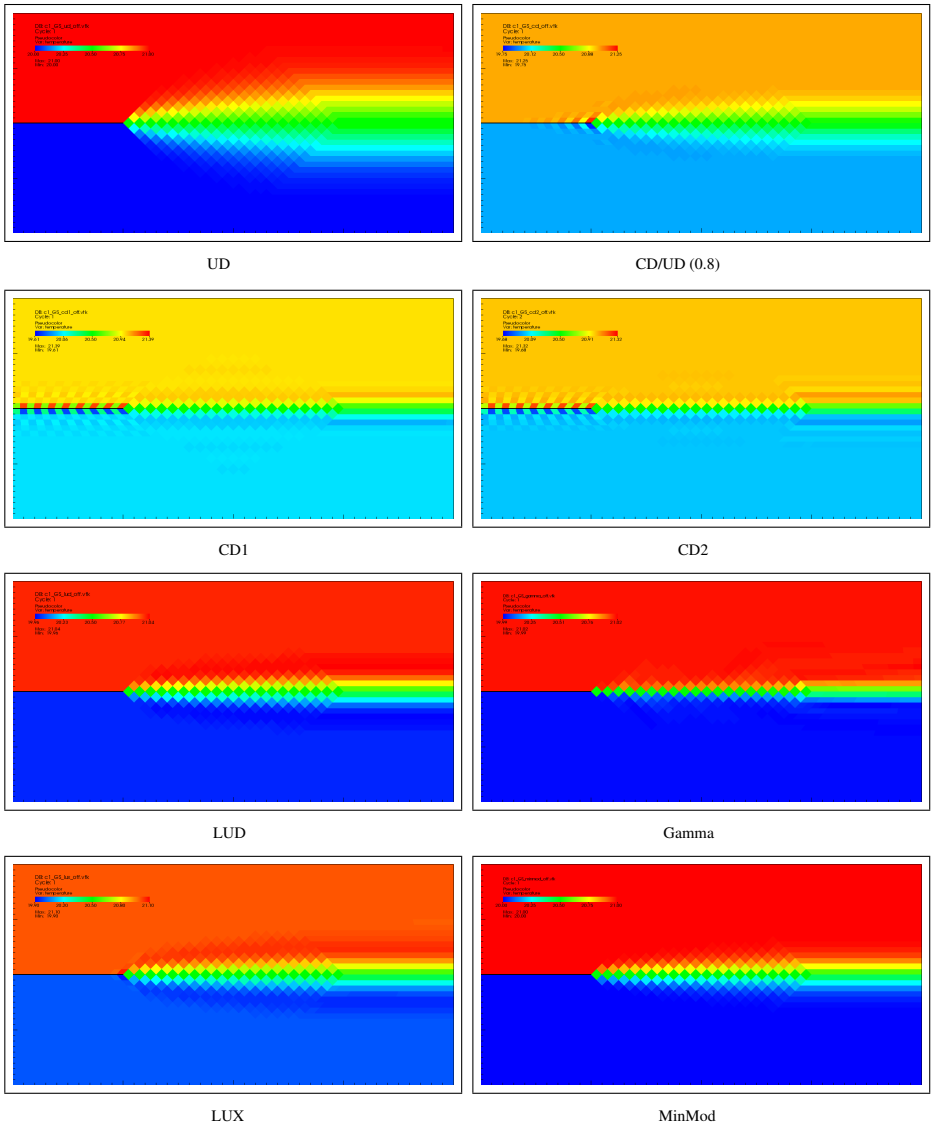


Figure 2.2: Standard case, all Gauss, no limiter

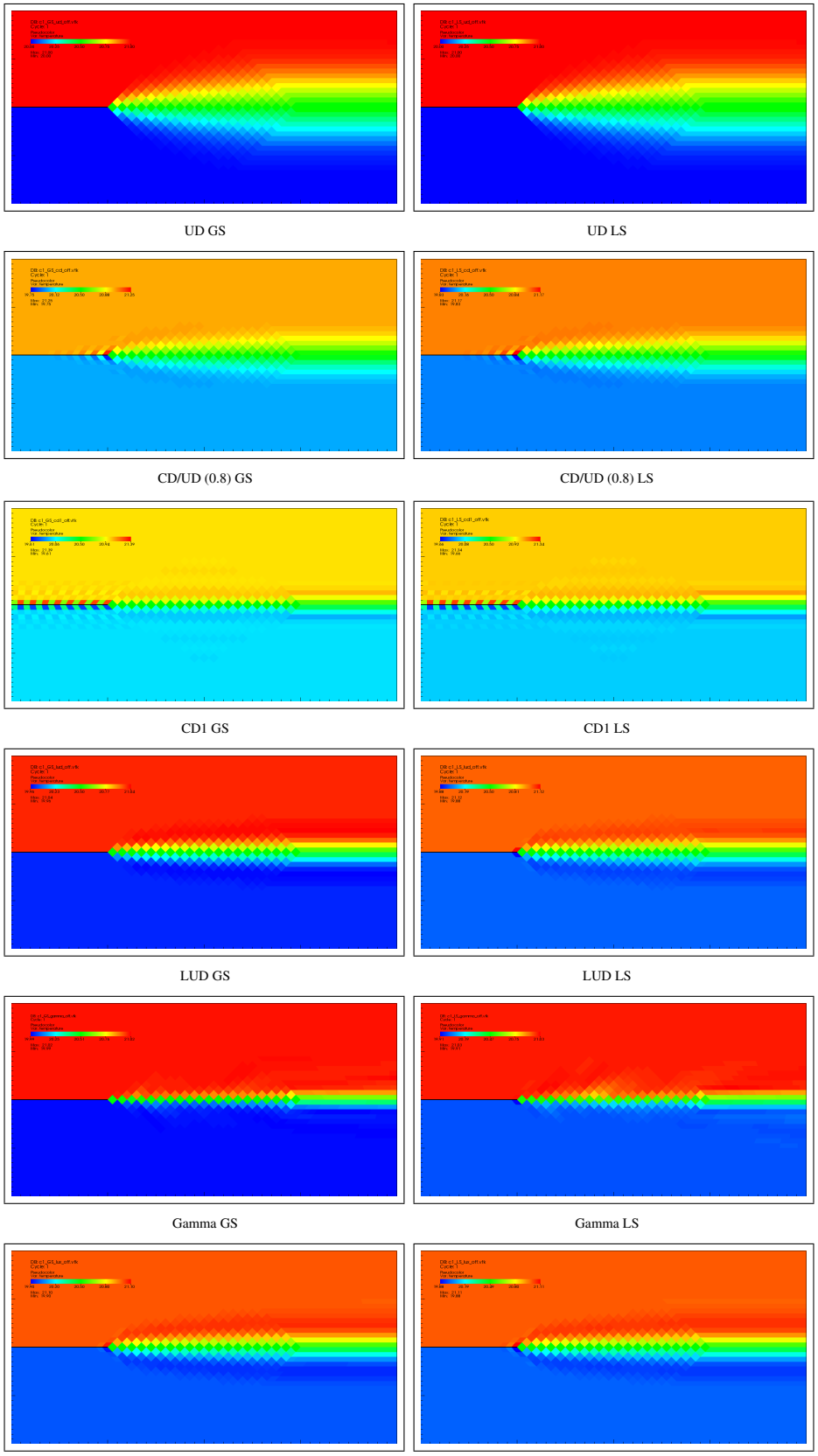
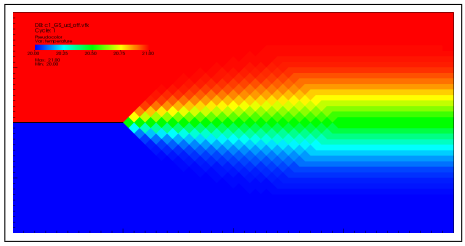
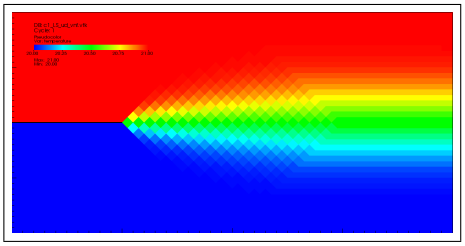


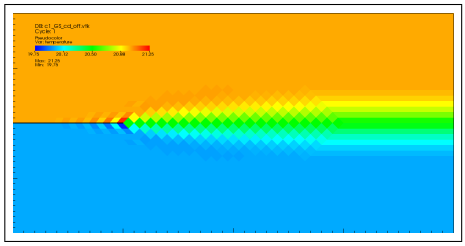
Figure 2.3: Standard case, left Gauss/right Least Squares, no limiter



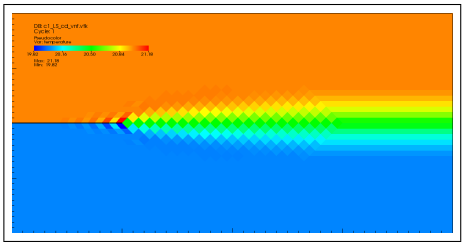
UD off



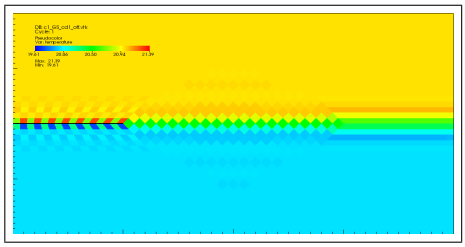
UD VNf



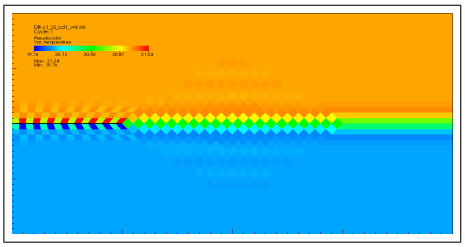
CD/UD (0.8) off



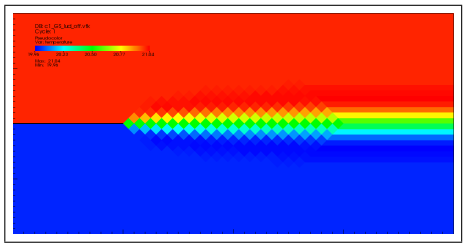
CD/UD (0.8) VNf



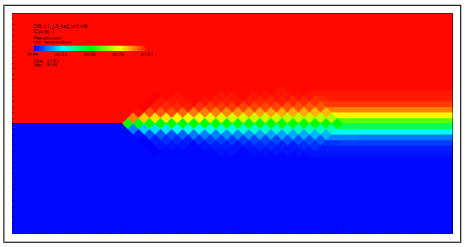
CD1 off



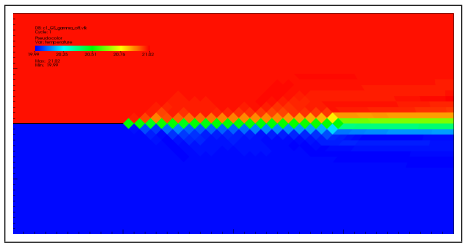
CD1 VNf



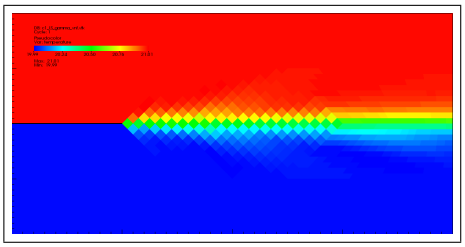
LUD off



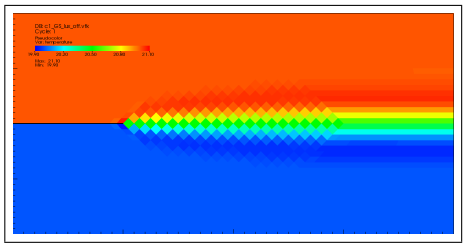
LUD VNf



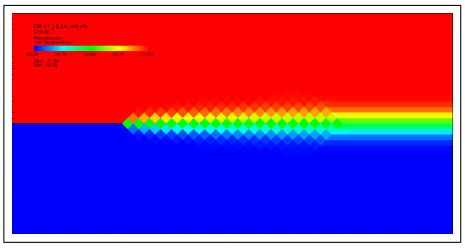
Gamma off



Gamma VNf



LUX off



LUX VNf

Figure 2.4: Standard case, Gauss, left no limiter/right VNf

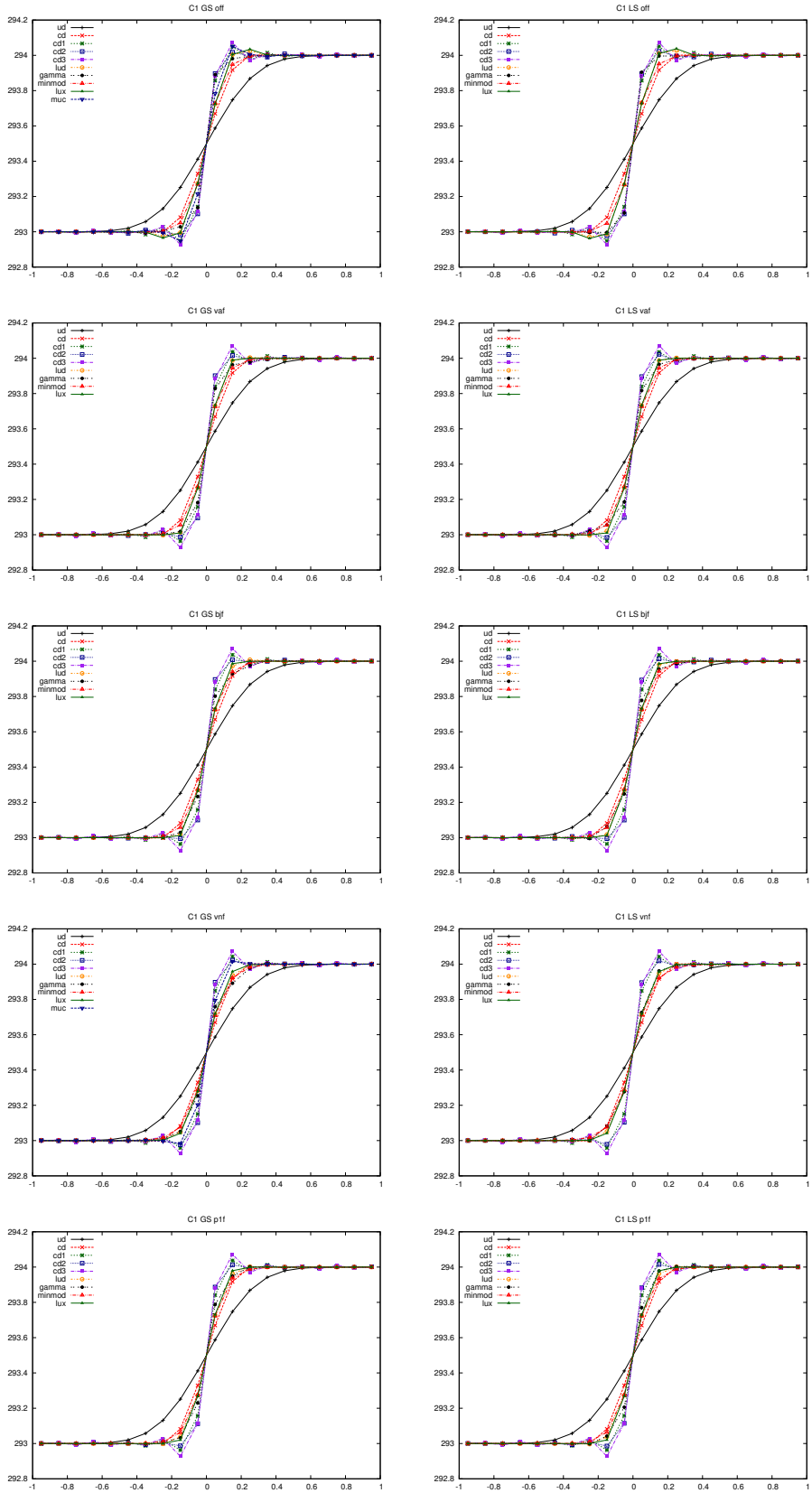


Figure 2.5: Standard case, profiles at x=1

3 C3 Leonard tests

Flow in a simple equidistant domain with 45 degrees which is the worst case for numerical diffusion. Three different profiles are considered:

step Very simple sudden step profile. The interface should remain sharp:

$$\phi(x) = \begin{cases} 20 & \text{for } 0 \leq x \leq \frac{1}{6}\ , \\ 21 & \text{for } \frac{1}{6} \leq x \leq 1\ . \end{cases} \tag{3.1}$$

sin2 The smooth varying $\sin^2$ profile (smooth begining and end, smooth maximum). The maximum should be preserved and not be clipped to a lower value:

$$\phi(x) = \begin{cases} 21\sin^2(3\pi(x - \frac{1}{6})) & \text{for } \frac{1}{6} \leq x \leq \frac{1}{2}\ , \\ 20 & \text{elsewhere.} \end{cases} \tag{3.2}$$

semi The semi ellipse which is basically a combination of the two previous profiles:

$$\phi(x) = \begin{cases} 21\sqrt{1 - (\frac{x-\frac{1}{3}}{\frac{1}{6}})^2)} & \text{for } \frac{1}{6} \leq x \leq \frac{1}{2}\ , \\ 20 & \text{elsewhere.} \end{cases} \tag{3.3}$$

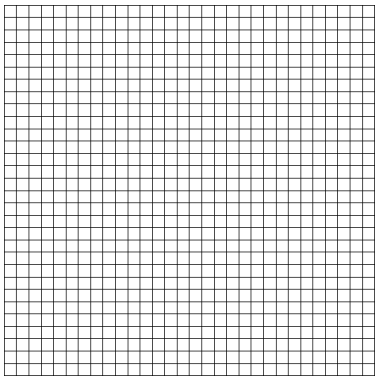


Figure 3.1: Mesh model c3

3.1 Step cases

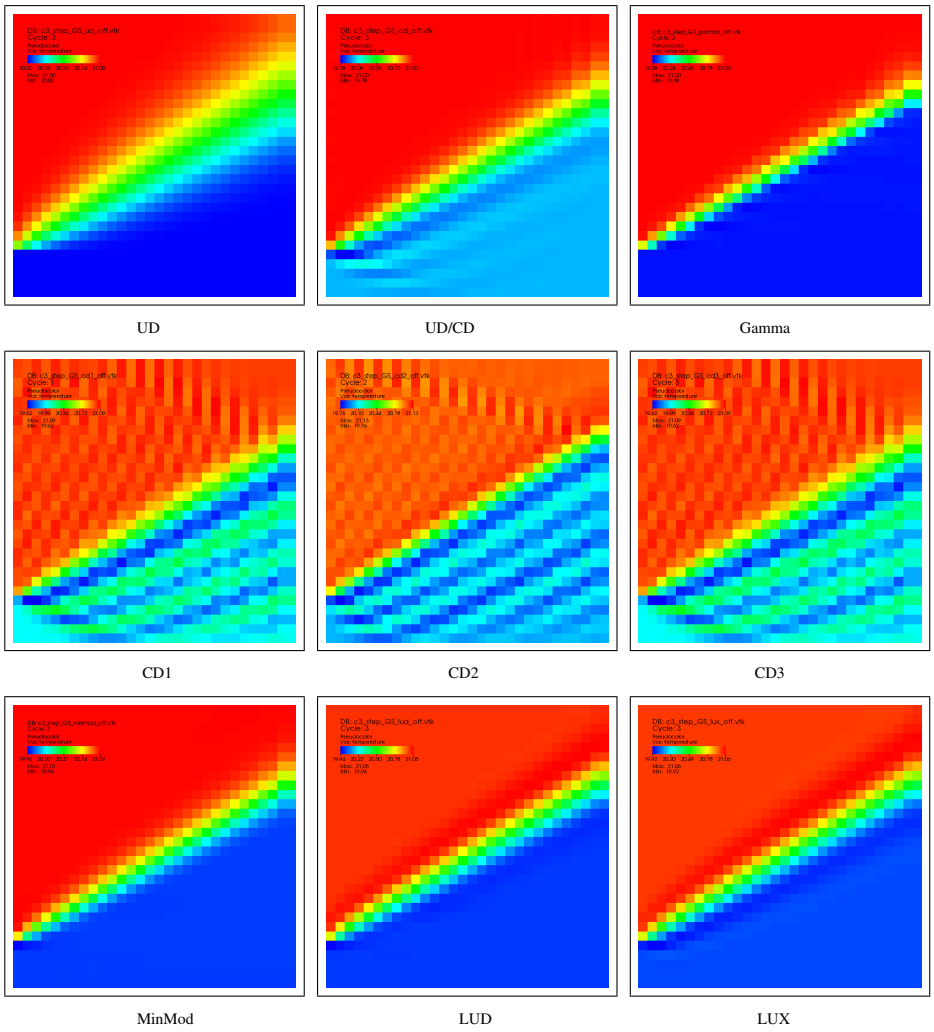


Figure 3.2: Standard Leonard step cases with Gauss, no limiter

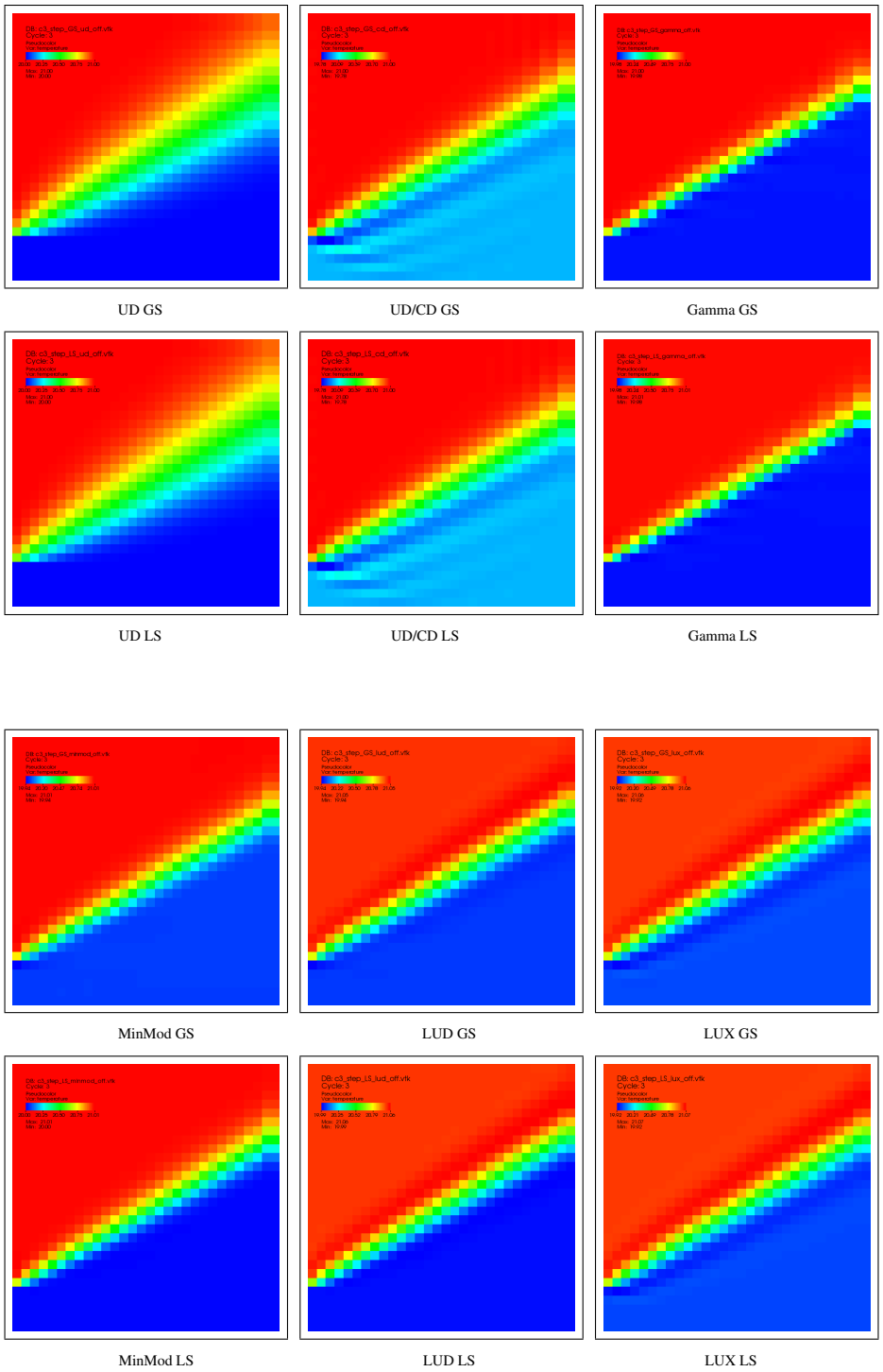


Figure 3.3: Standard Leonard step cases with Gauss (GS) and Least Squares (LS), no limiters used

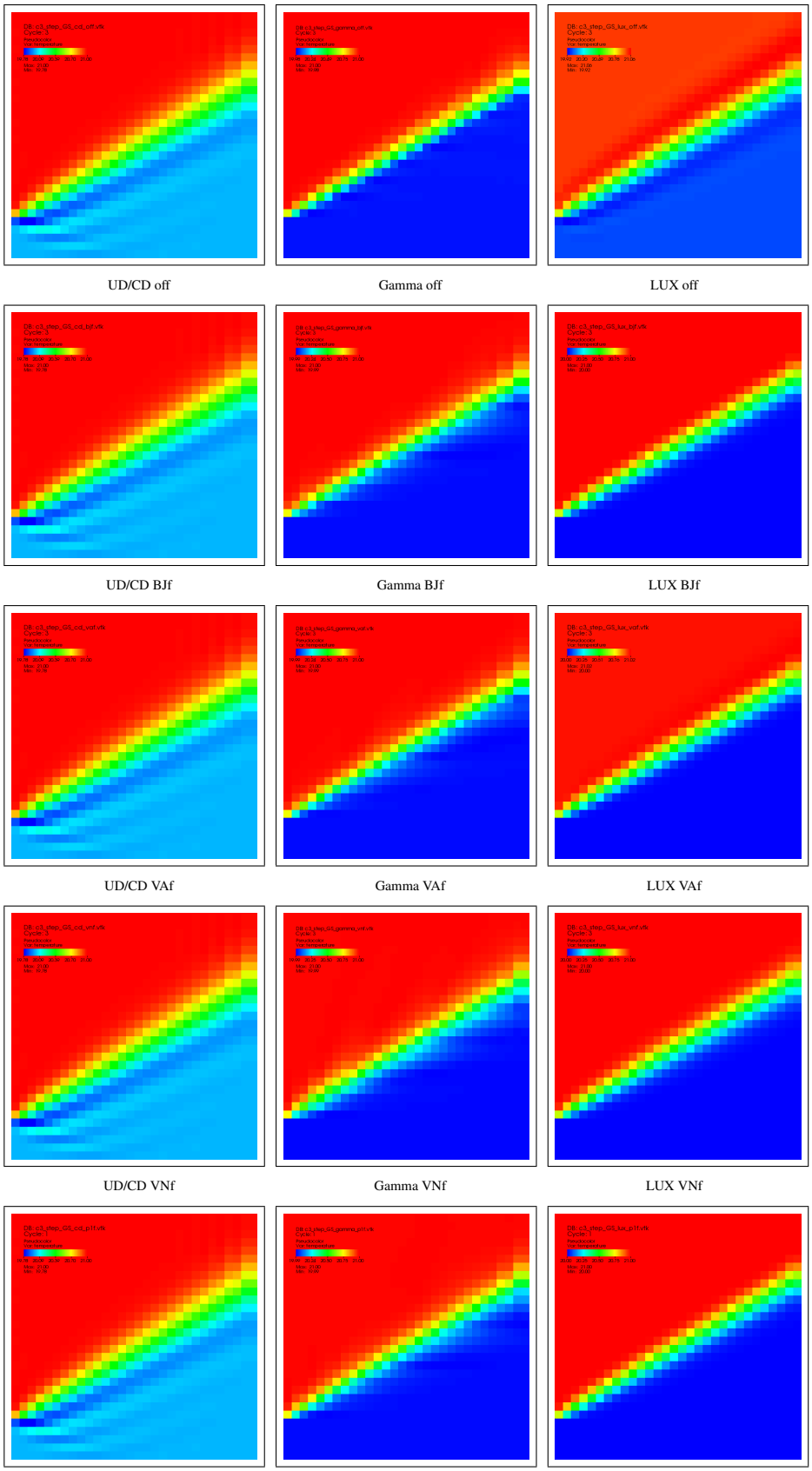


Figure 3.4: Leonard step cases with Gauss and face based limiters

3.2 Sin2 cases

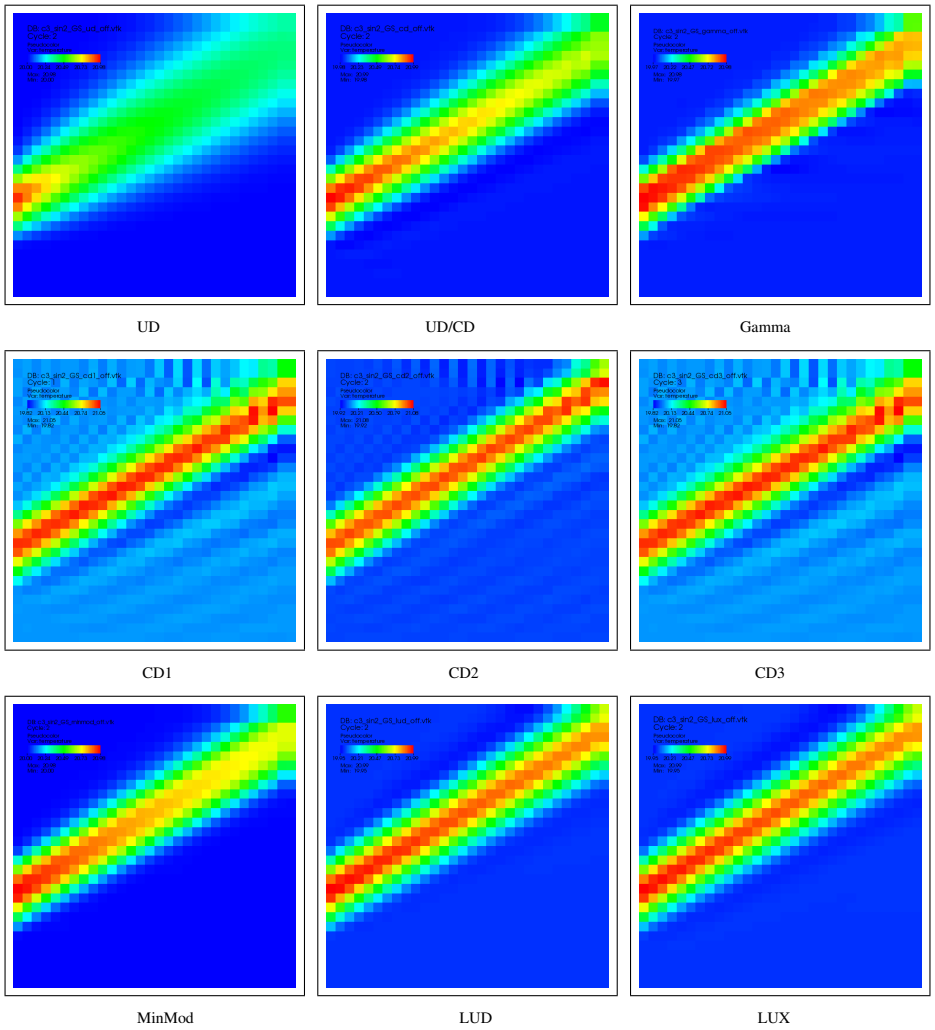


Figure 3.5: Standard Leonard sin2 cases with Gauss, no limiter

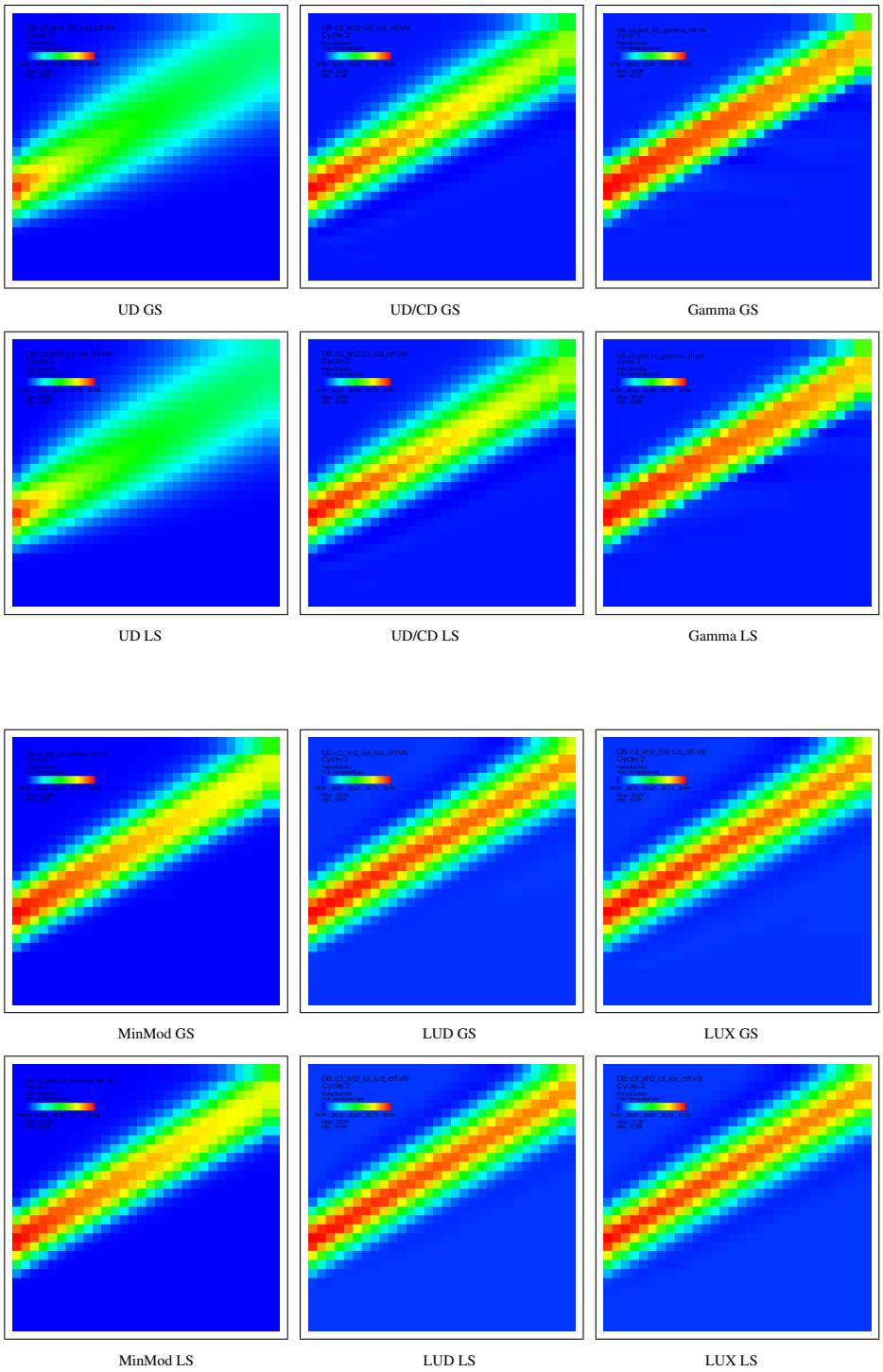


Figure 3.6: Standard Leonard sin2 cases with Gauss (GS) and Least Squares (LS), no limiters used

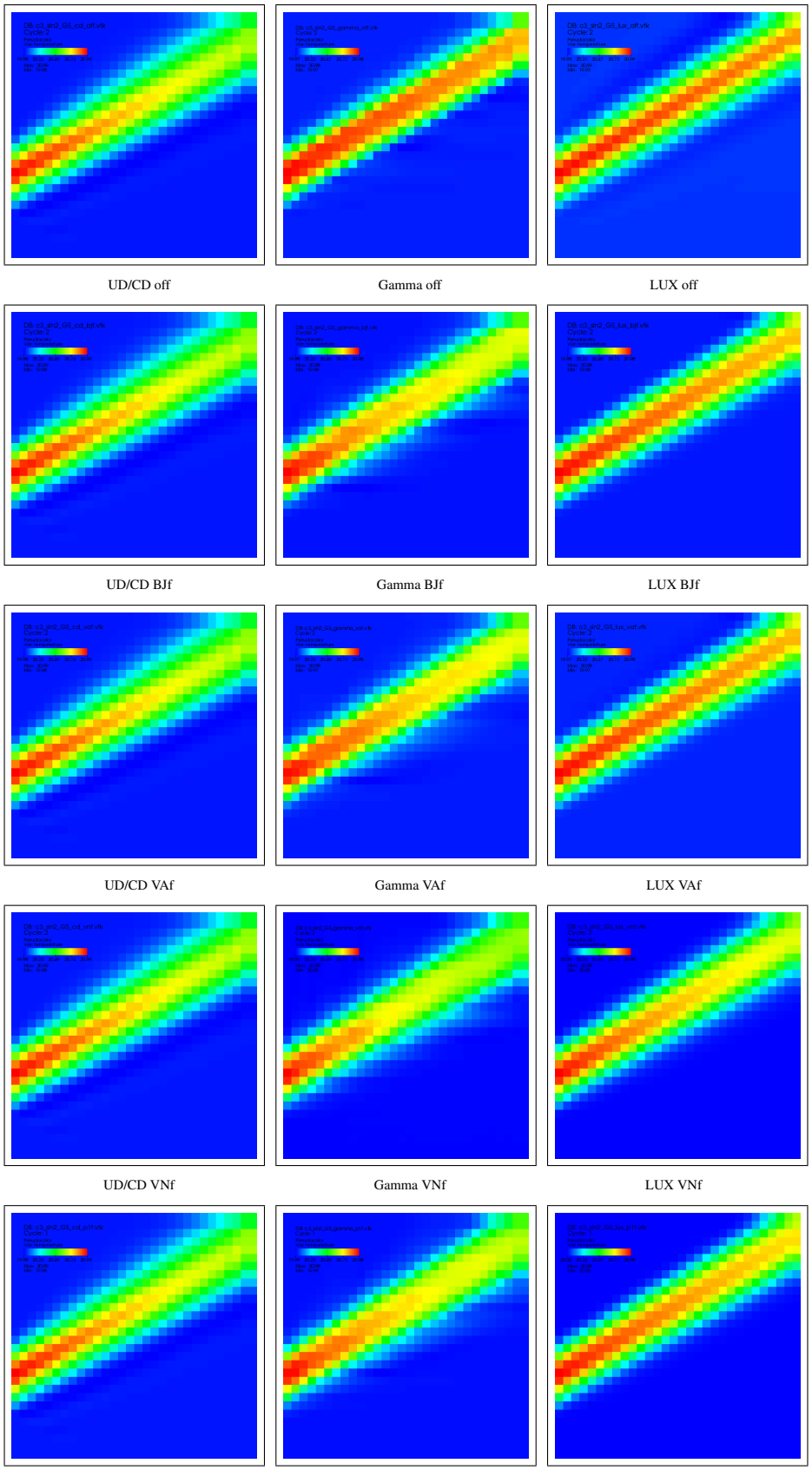


Figure 3.7: Leonard sin2 cases with Gauss and face based limiters

3.3 Semi ellipse cases

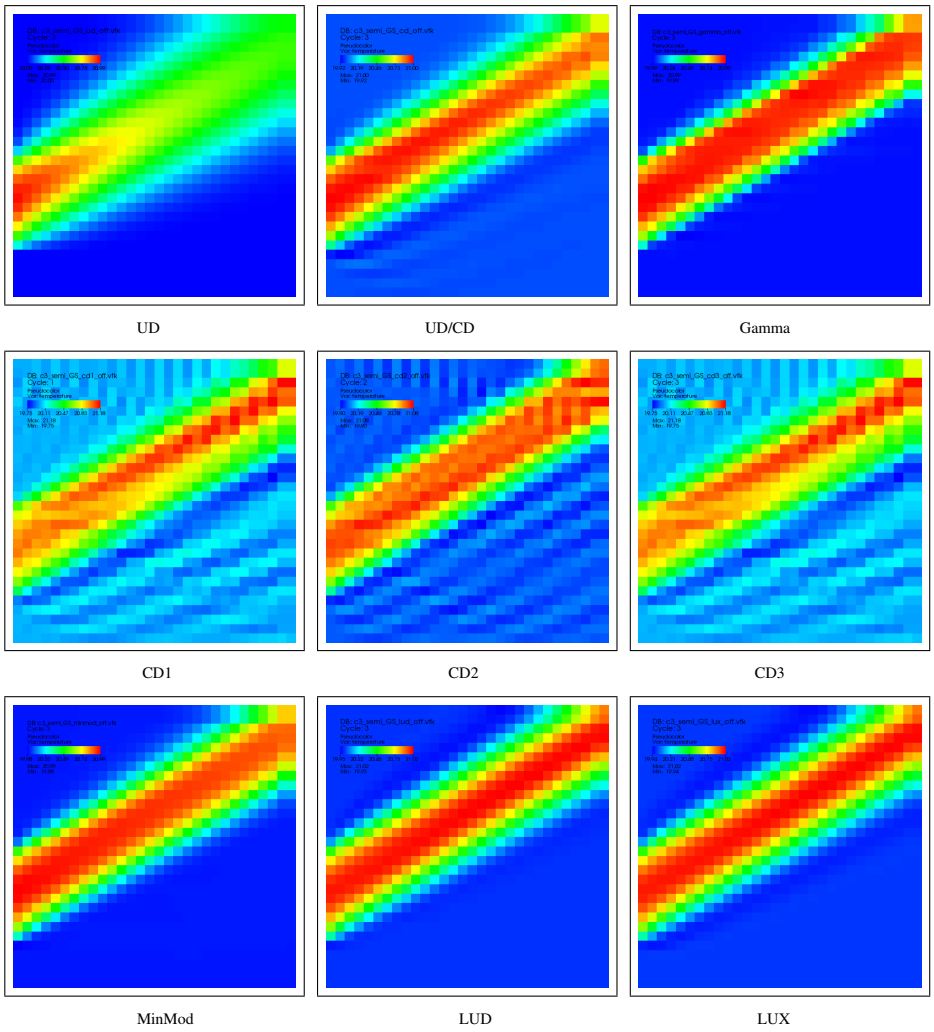


Figure 3.8: Standard Leonard semi cases with Gauss, no limiter

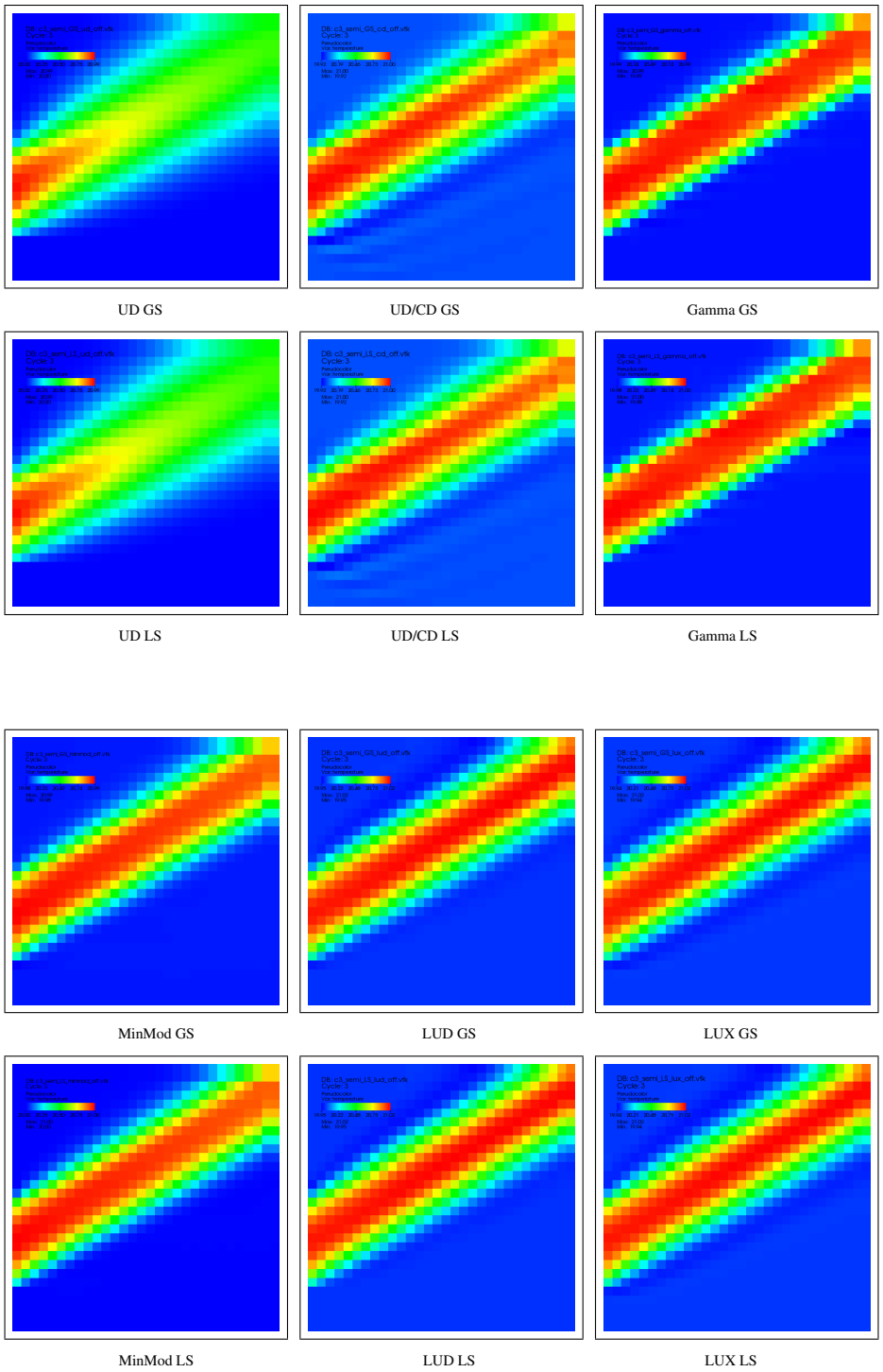


Figure 3.9: Standard Leonard semi cases with Gauss (GS) and Least Squares (LS), no limiters used

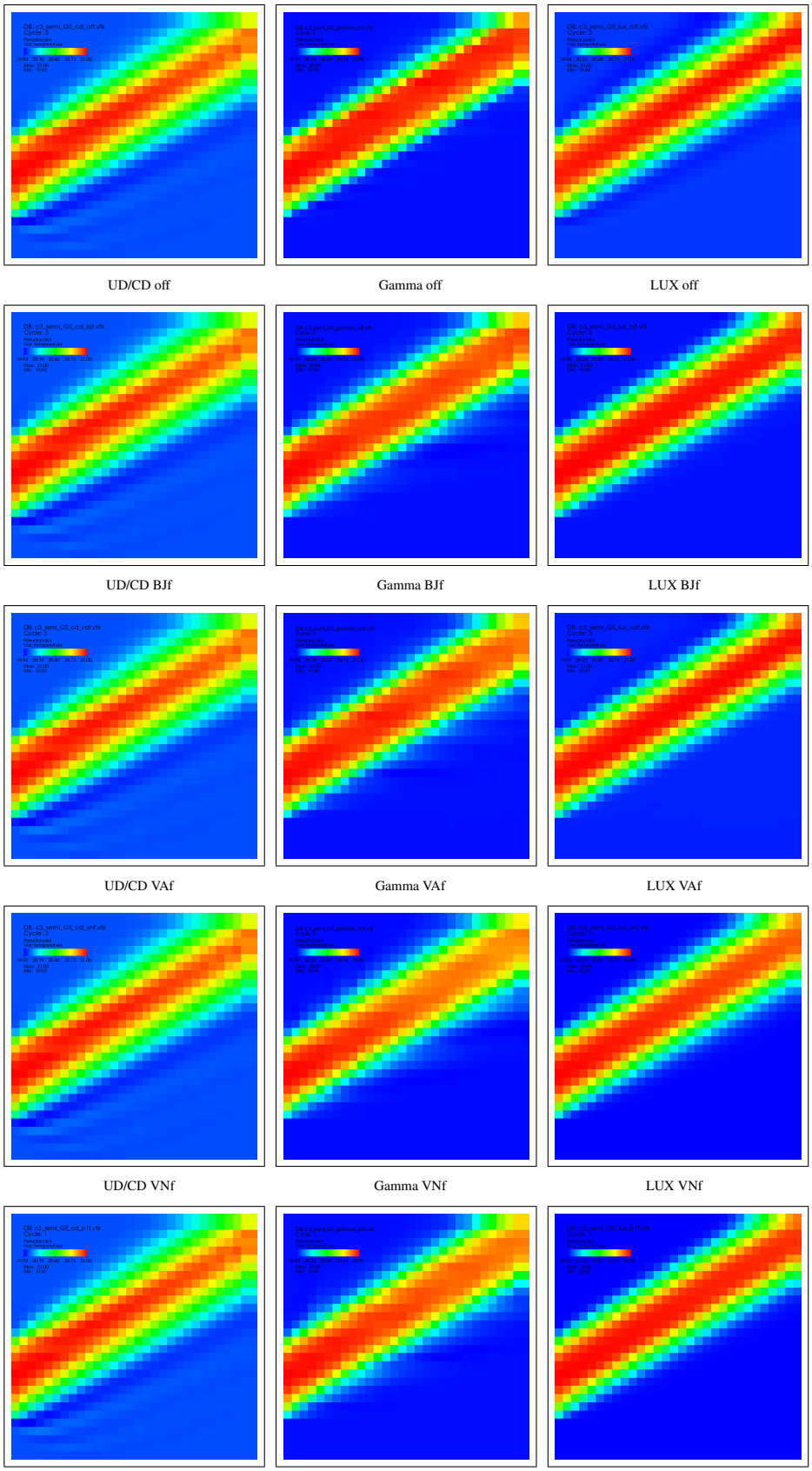


Figure 3.10: Leonard semi cases with Gauss and face based limiters

4 C4 Wild test

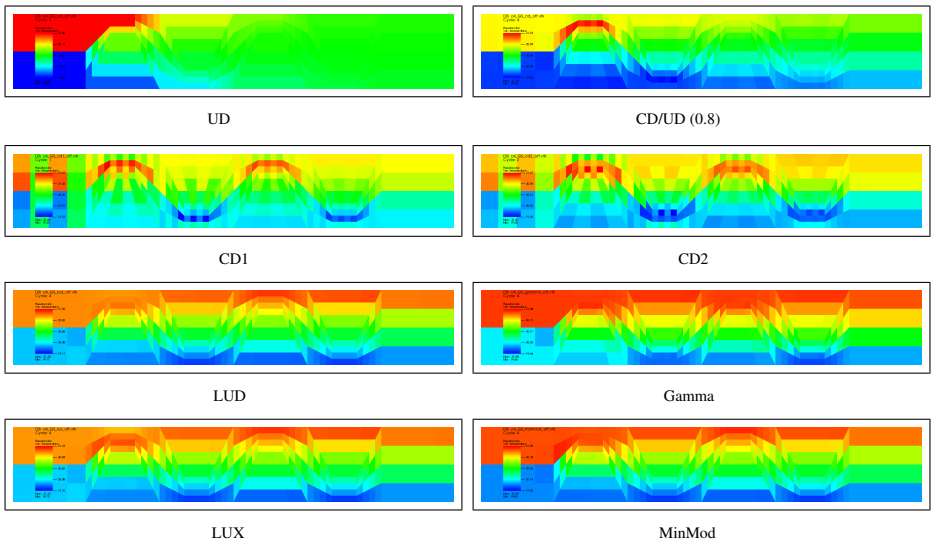


Figure 4.1: Wild case, all Gauss, no limiter

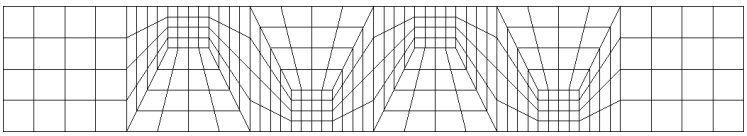


Figure 4.2: Mesh model c4

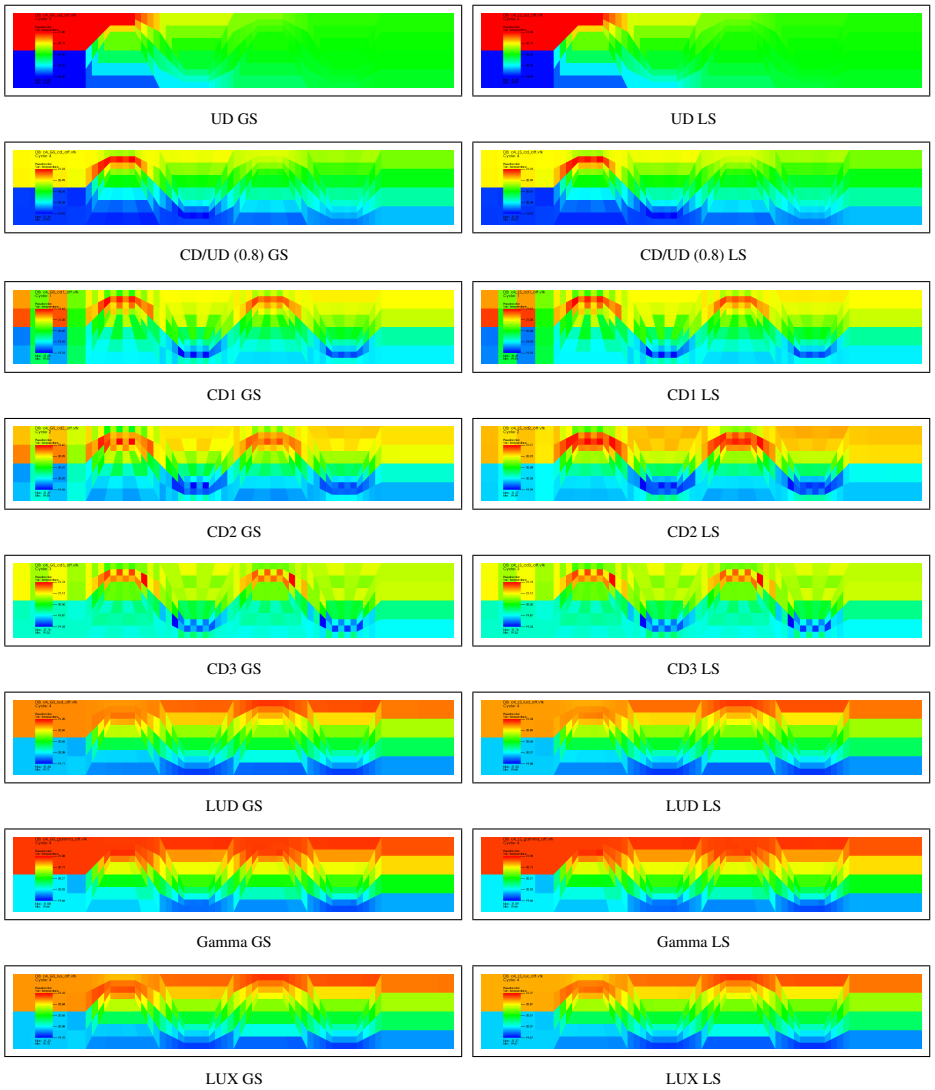


Figure 4.3: Wild case, left Gauss/right Least Squares, no limiters

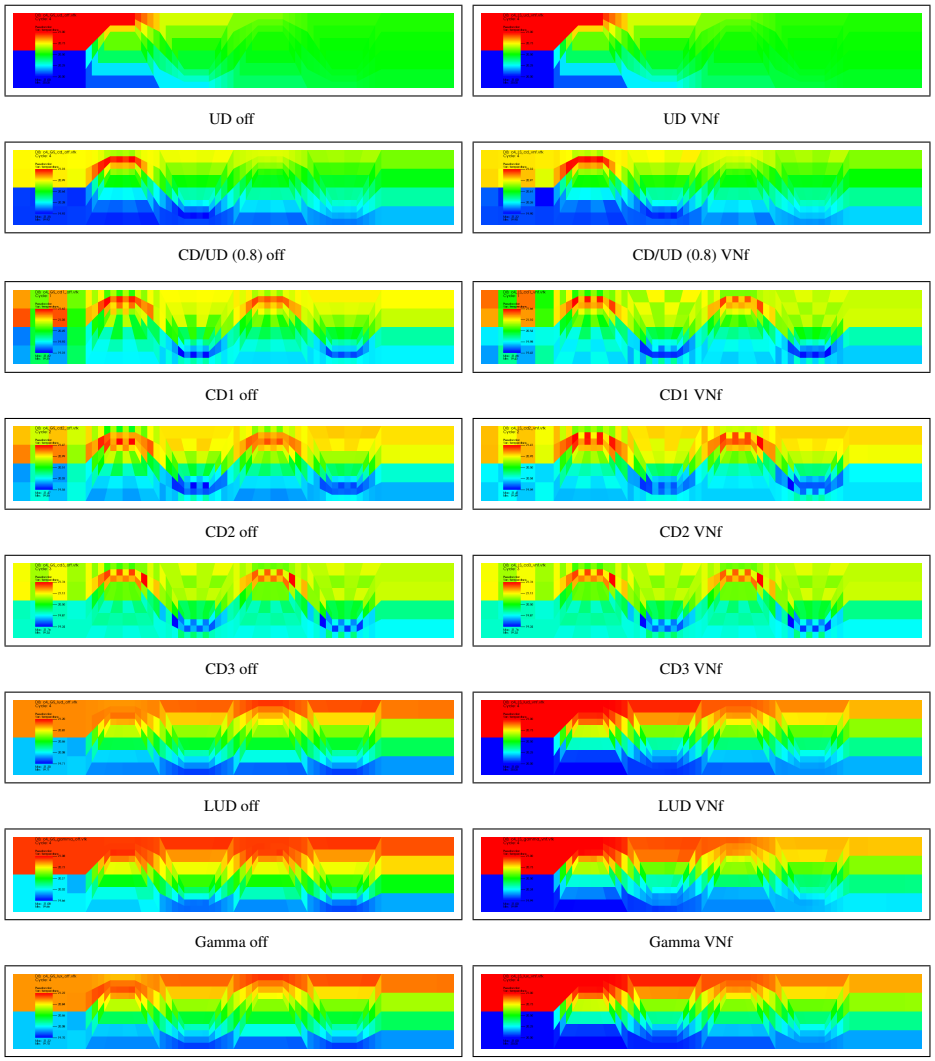


Figure 4.4: Wild case, Gauss, left no limiter/right VNf

5 C6 Wedges

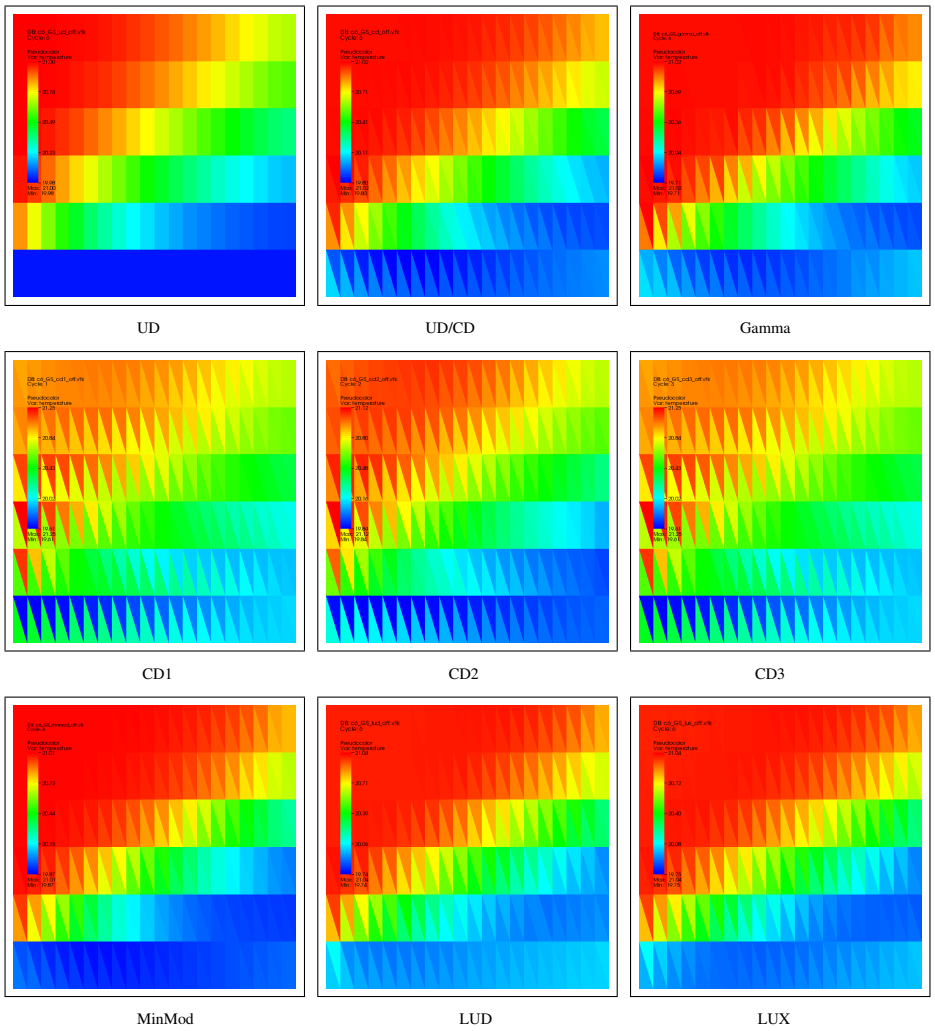


Figure 5.1: c6 standard wedge cases with Gauss, no limiter

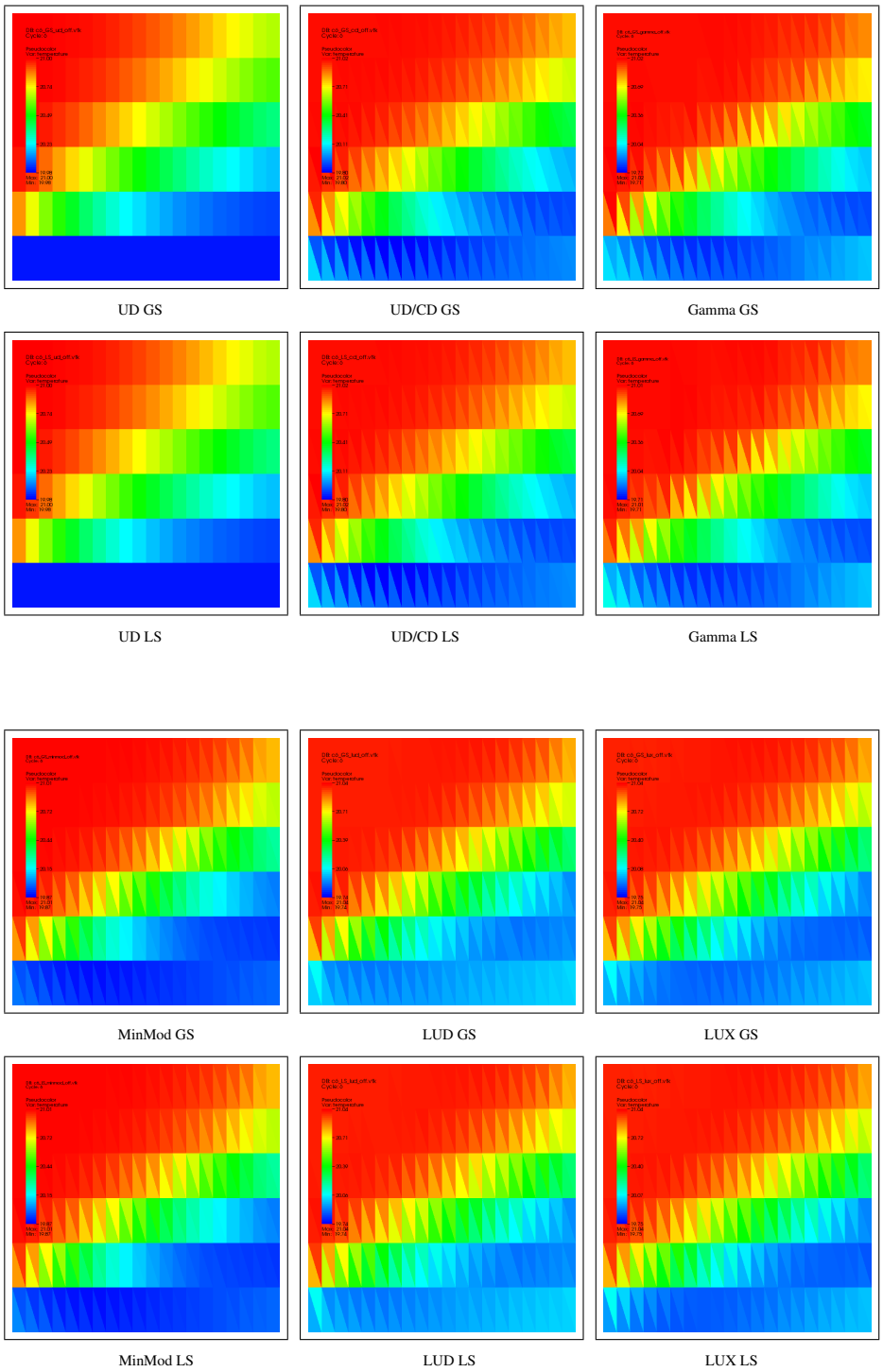


Figure 5.2: c6 standard wedge cases with Gauss (GS) and Least Squares (LS), no limiters used

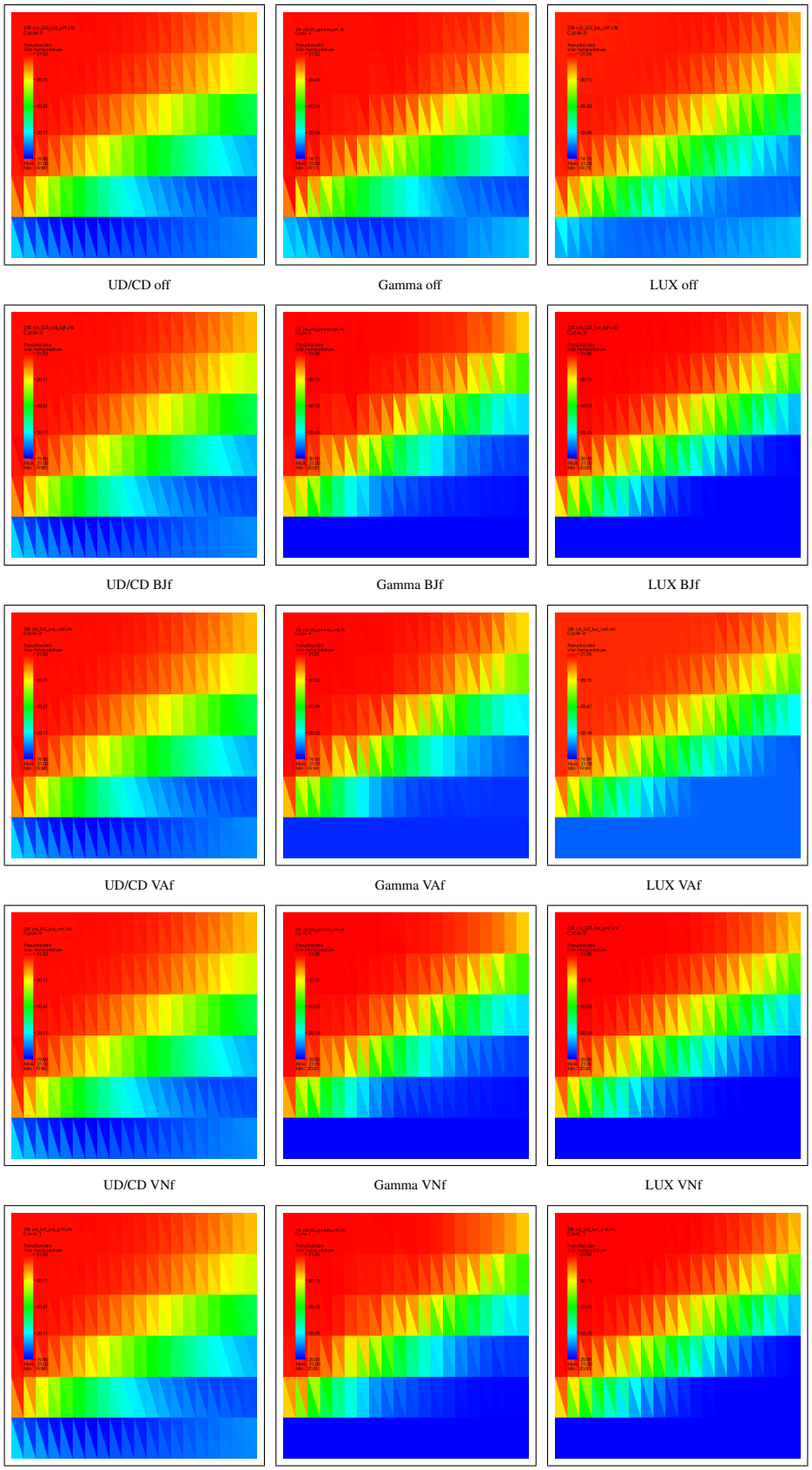


Figure 5.3: c6 wedge cases with Gauss and face based limiters

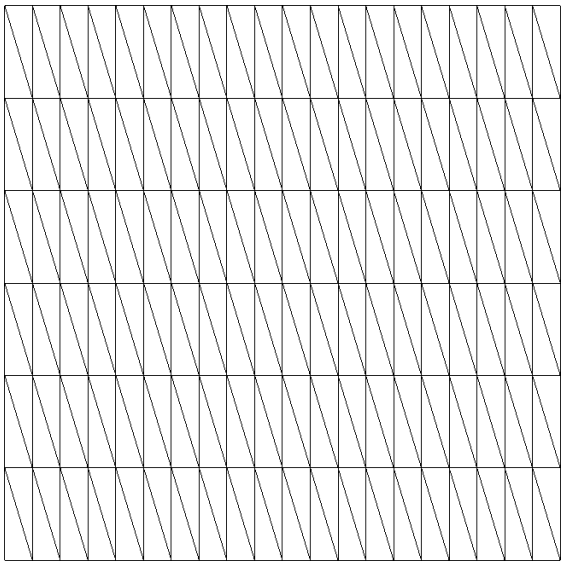


Figure 5.4: Mesh model c6

6 C7 Mesh jump

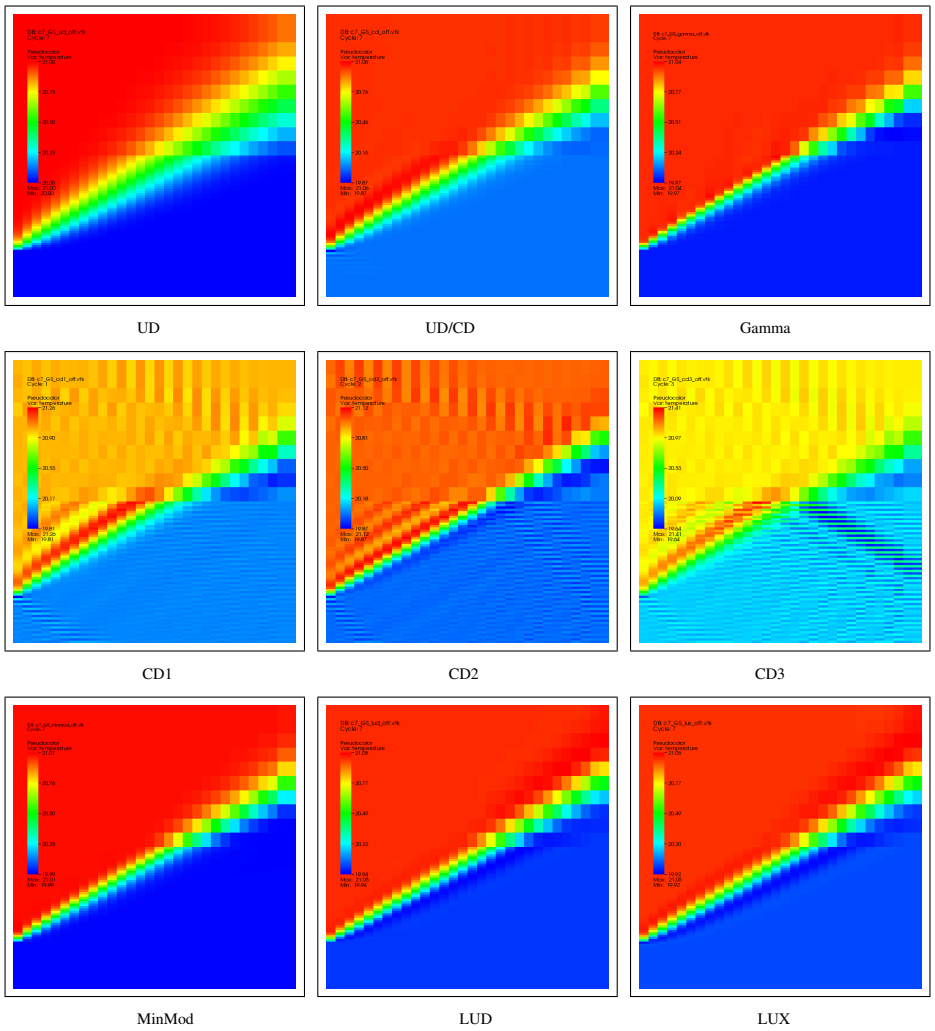


Figure 6.1: c7 mesh jump cases with Gauss, no limiter

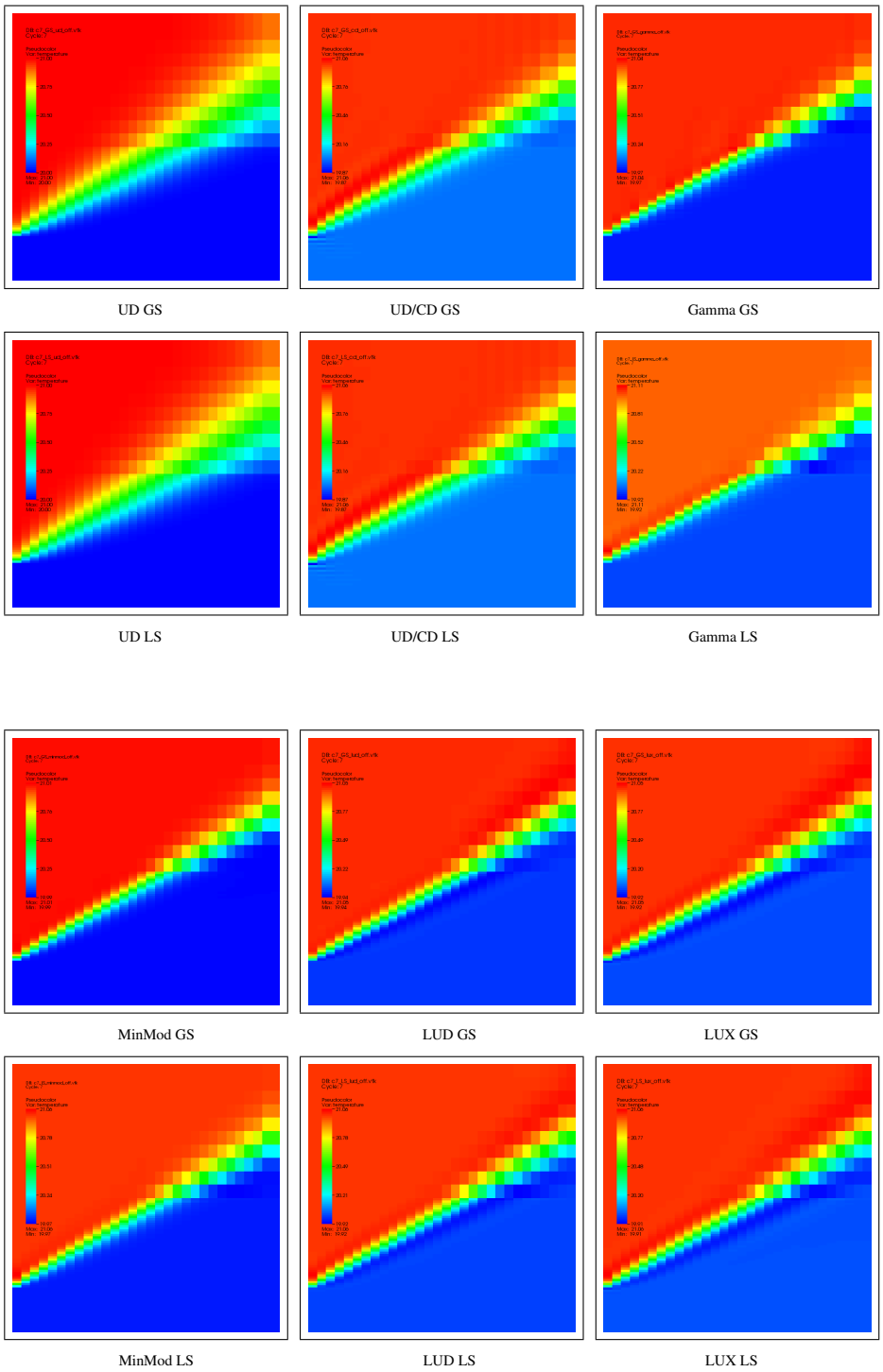


Figure 6.2: c7 mesh jump cases with Gauss (GS) and Least Squares (LS), no limiters used

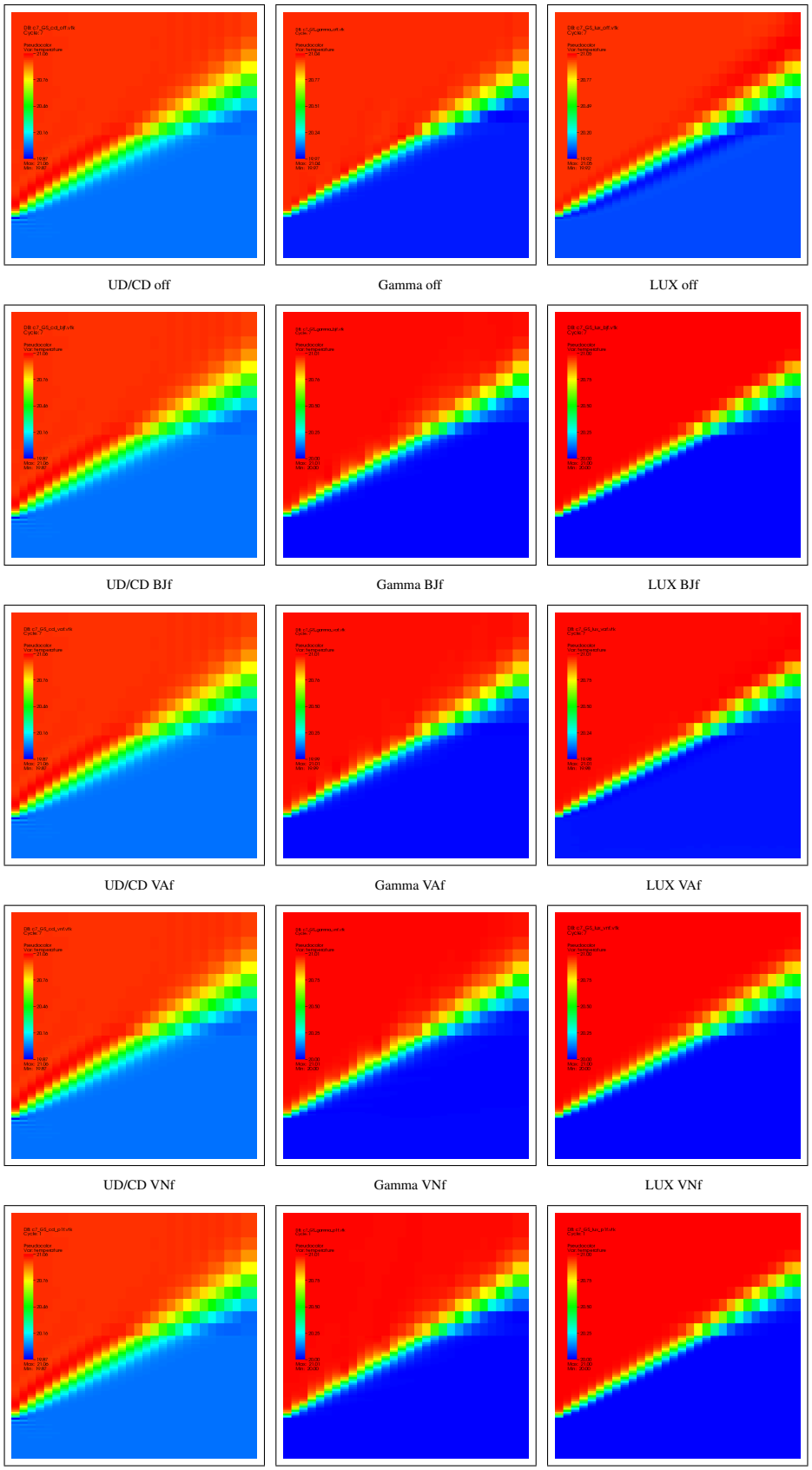


Figure 6.3: c7 mesh jump cases with Gauss and face based limiters

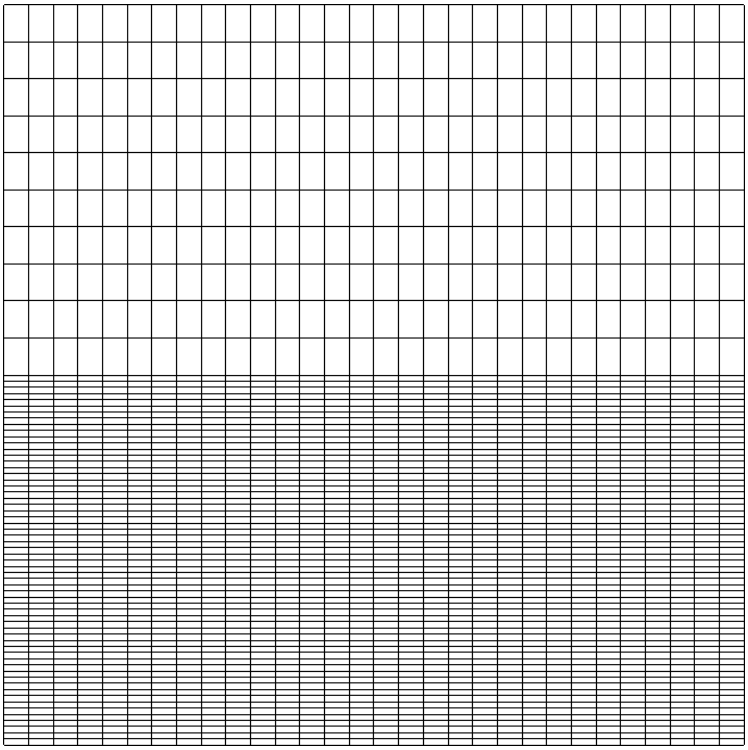


Figure 6.4: Mesh model c7

7 C8 Embedded refinement

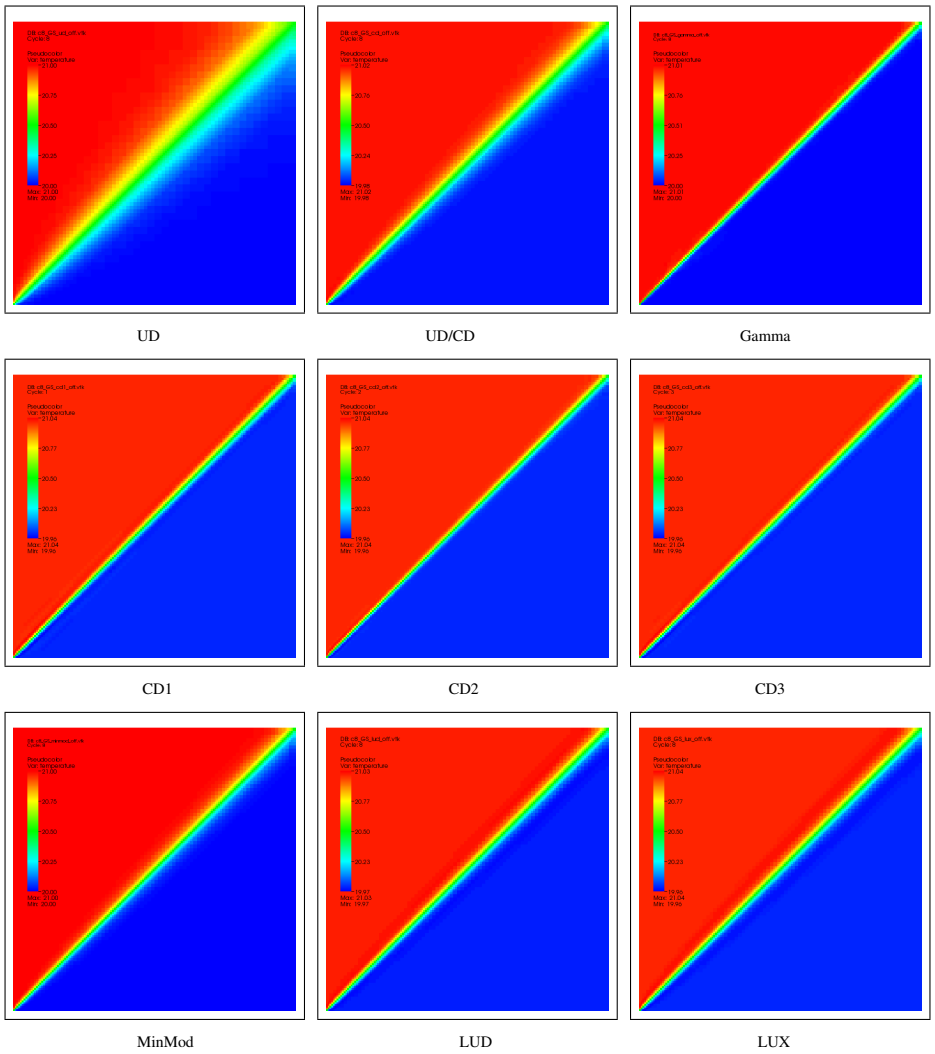


Figure 7.1: c8 refinement cases with Gauss, no limiter

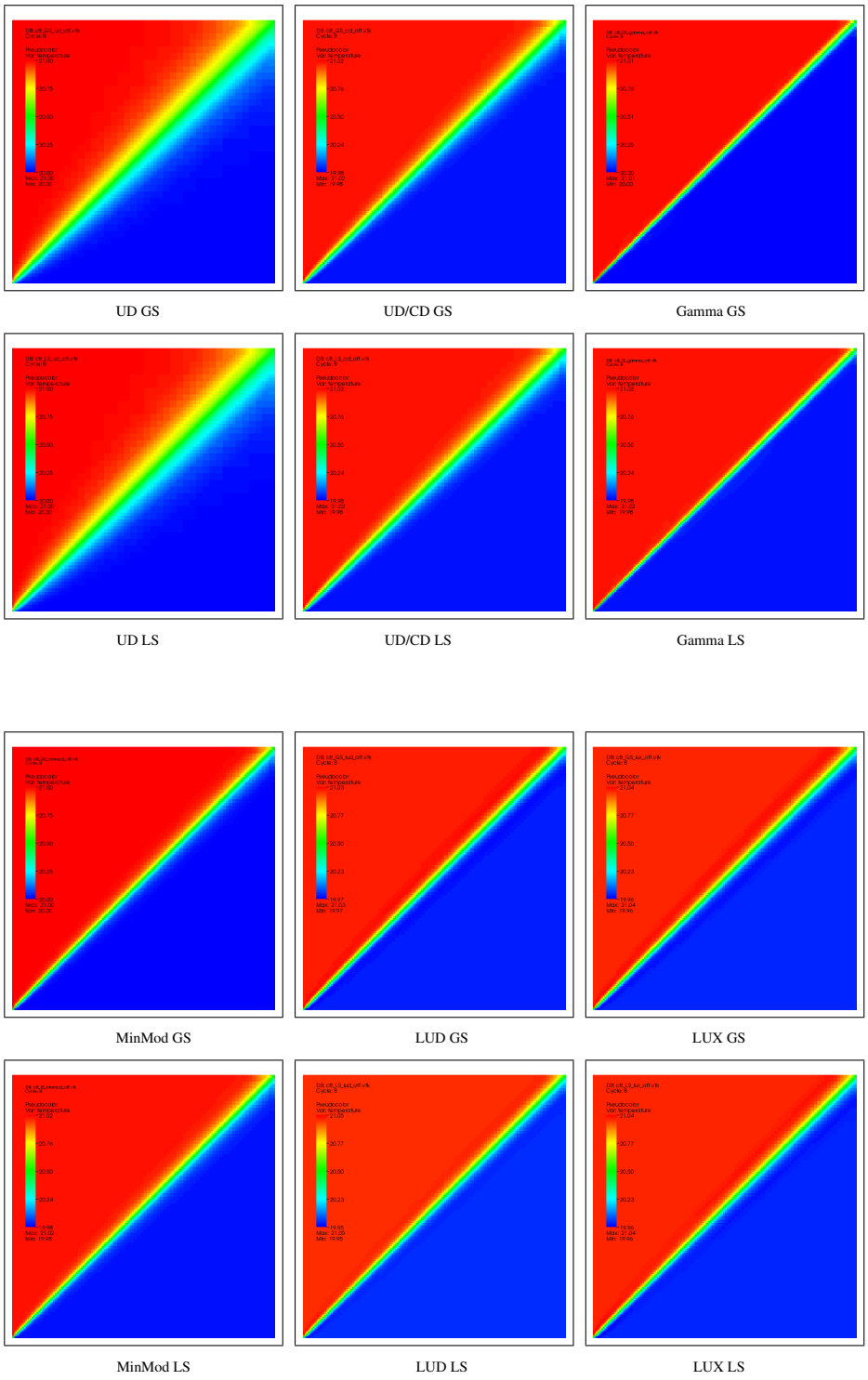


Figure 7.2: c8 refinement cases with Gauss (GS) and Least Squares (LS), no limiters used

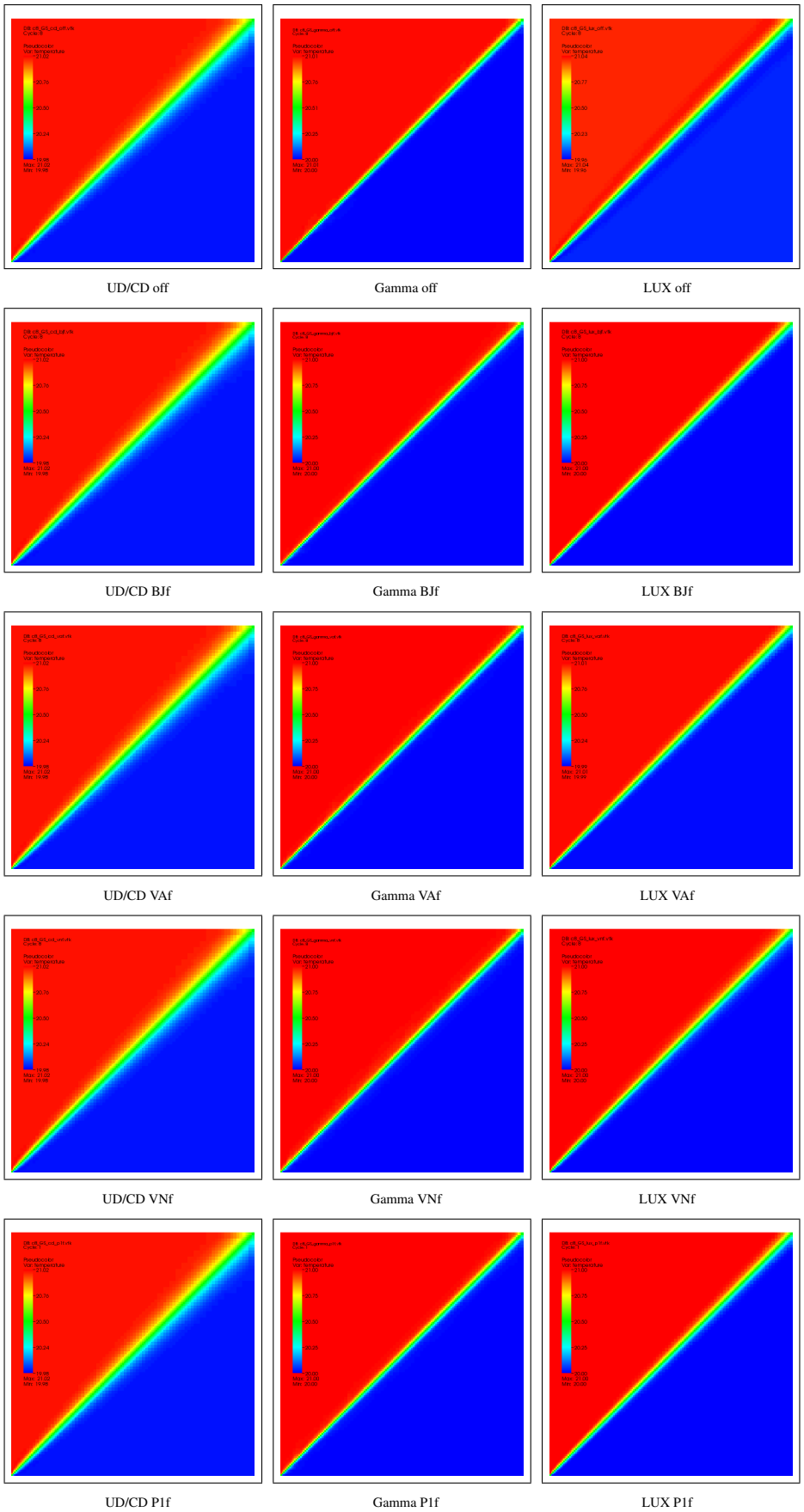


Figure 7.3: c8 refinement cases with Gauss and face based limiters

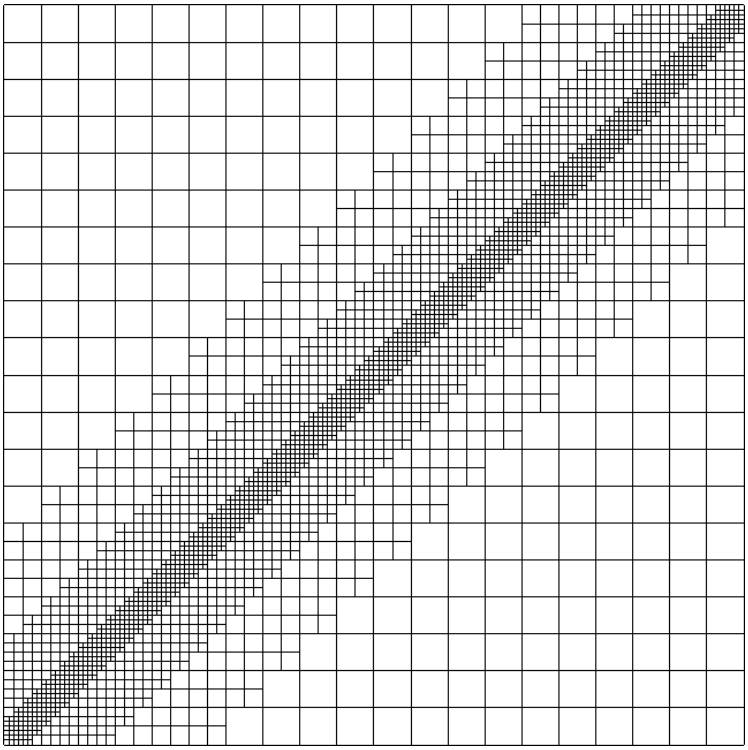


Figure 7.4: Mesh model c8

8 M6 Stagnation flow with embedded refinement

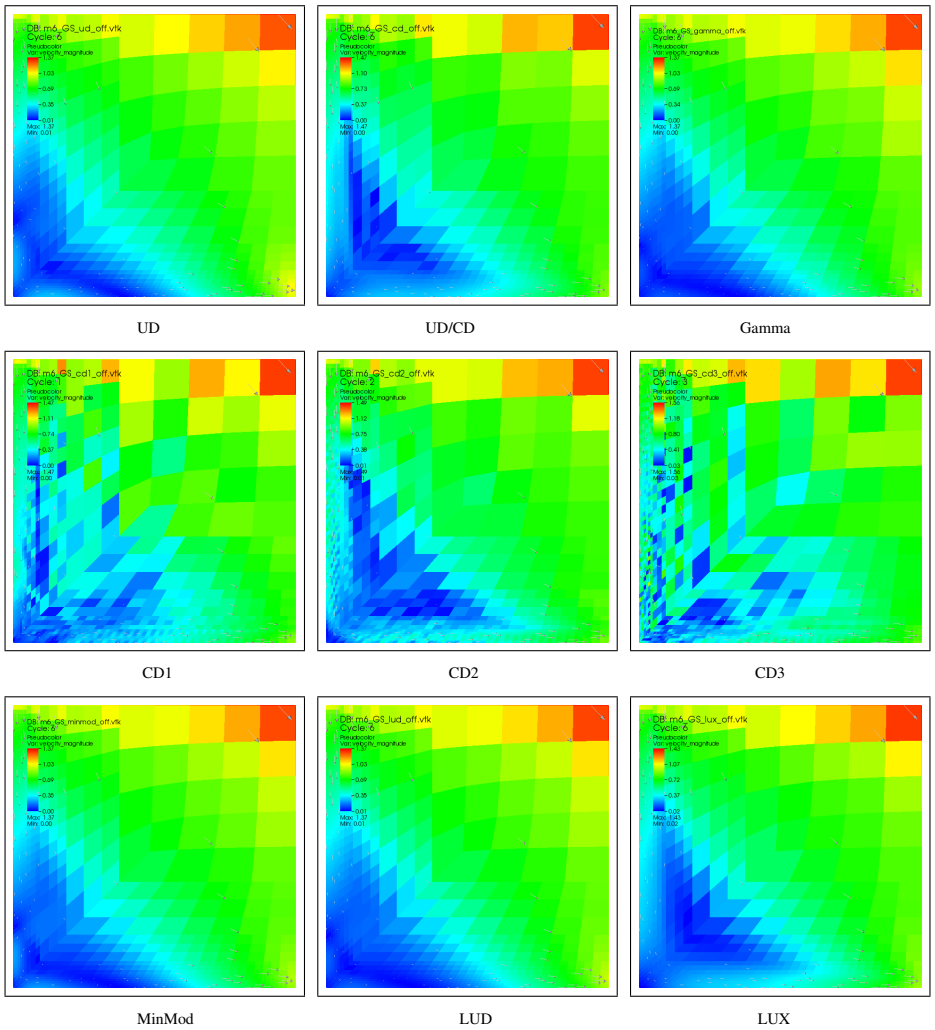


Figure 8.1: m6 refinement cases with Gauss, no limiter

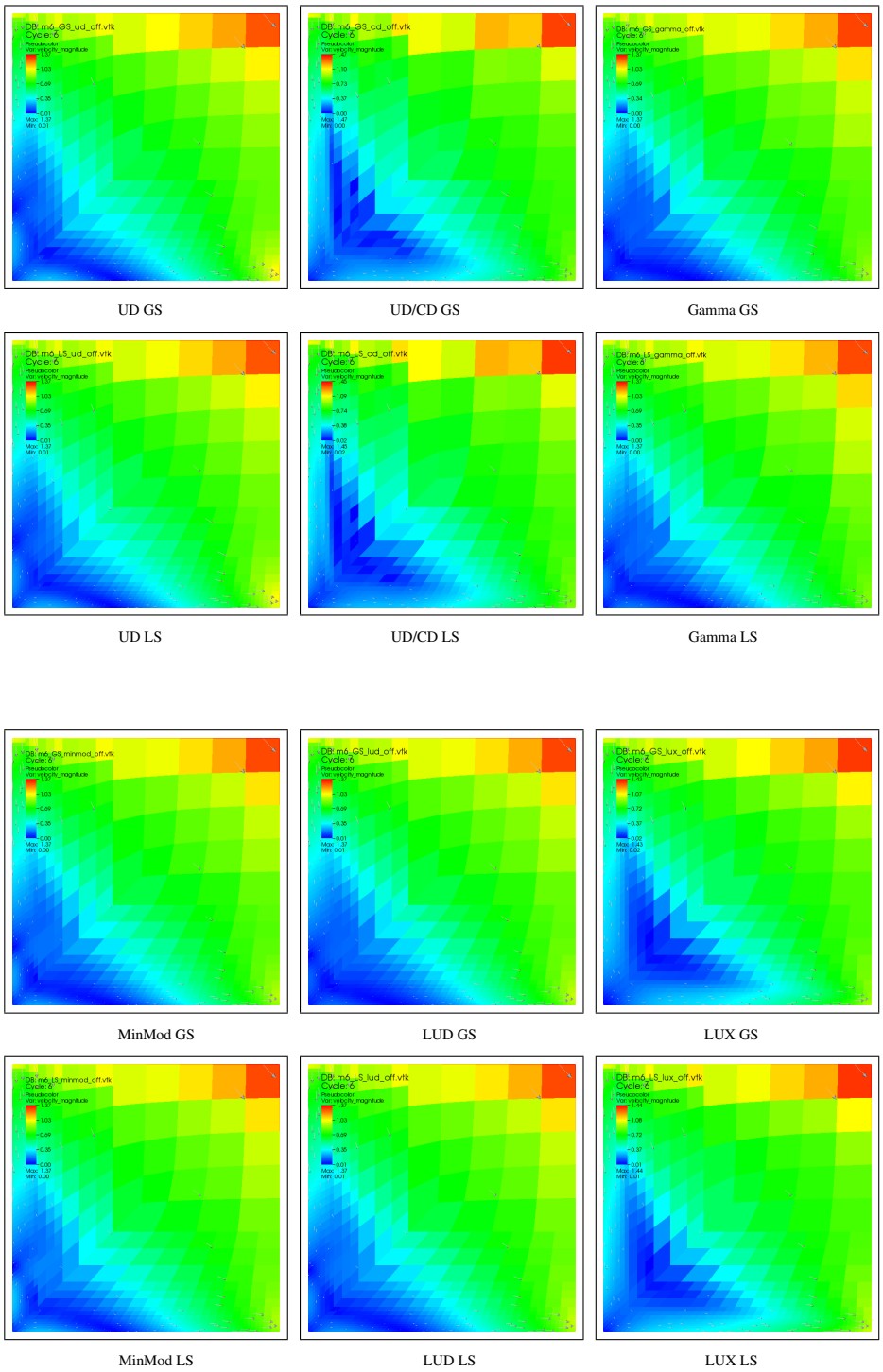


Figure 8.2: m6 refinement cases with Gauss (GS) and Least Squares (LS), no limiters used

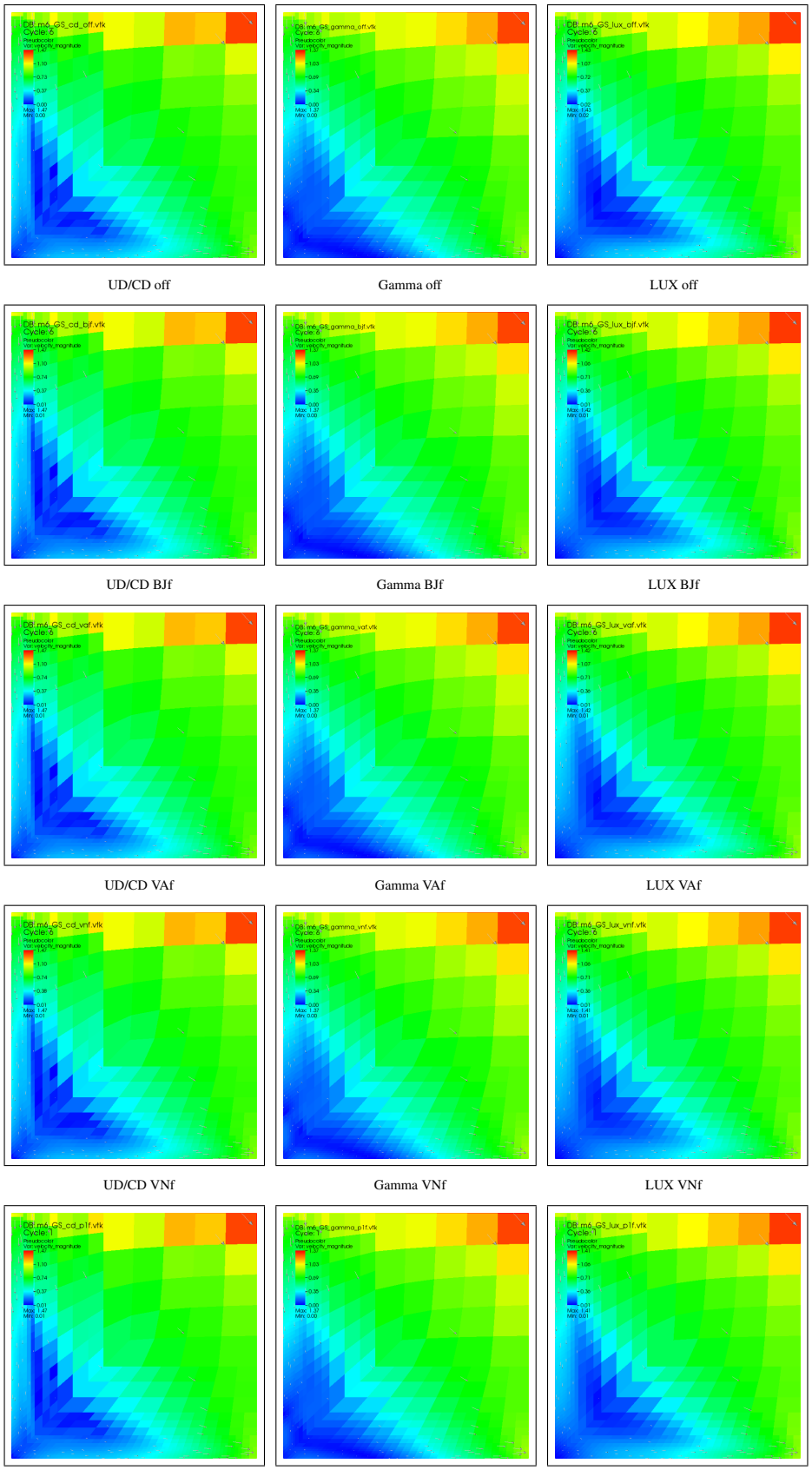


Figure 8.3: m6 refinement cases with Gauss and face based limiters

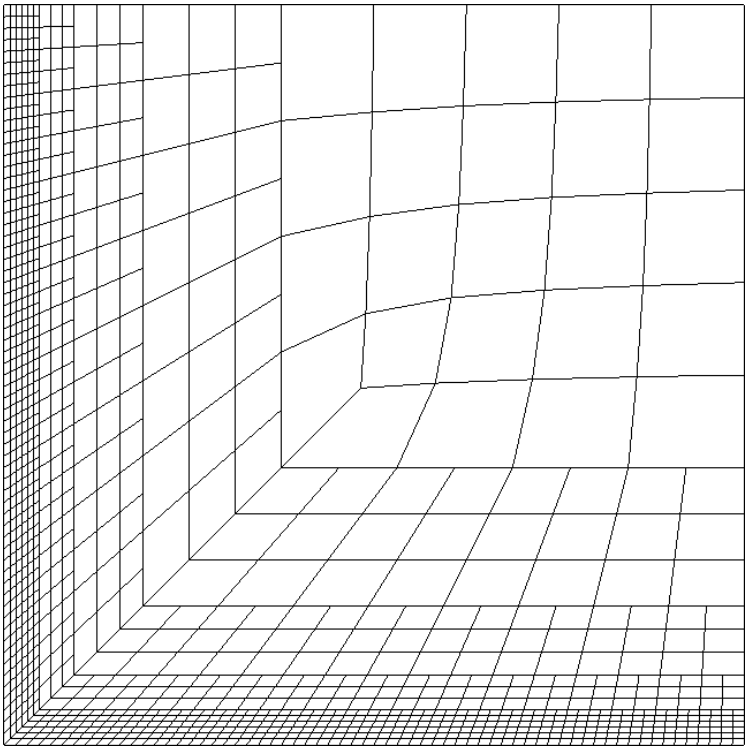


Figure 8.4: Mesh model m6

9 CAV Lid driven cavities

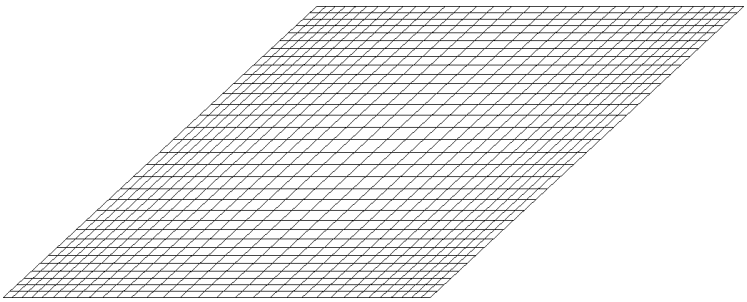


Figure 9.1: Mesh model cav45

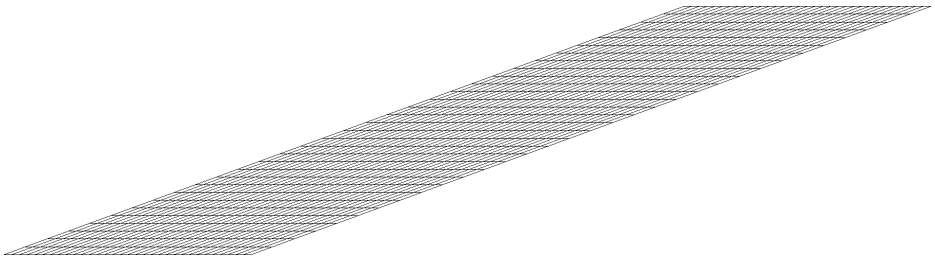


Figure 9.2: Mesh model cav20

9.1 Hexahedra at 45 degrees

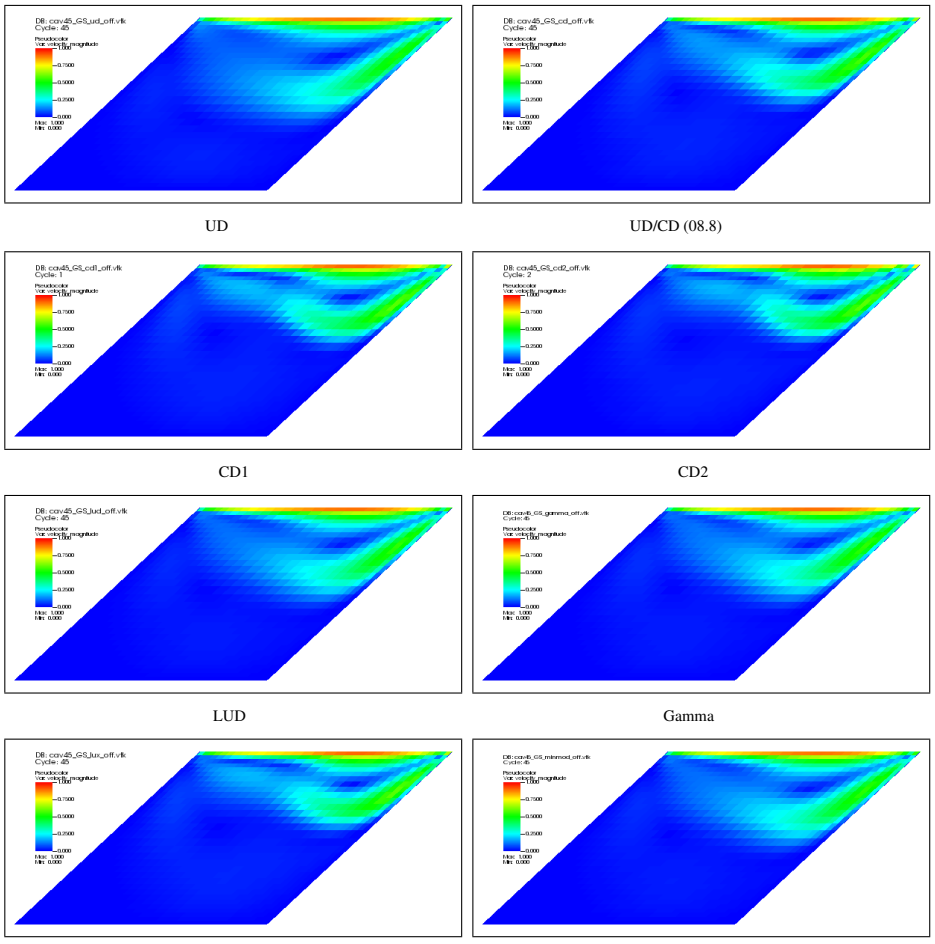


Figure 9.3: cav45 LDC at 45 degrees with Gauss, no limiter

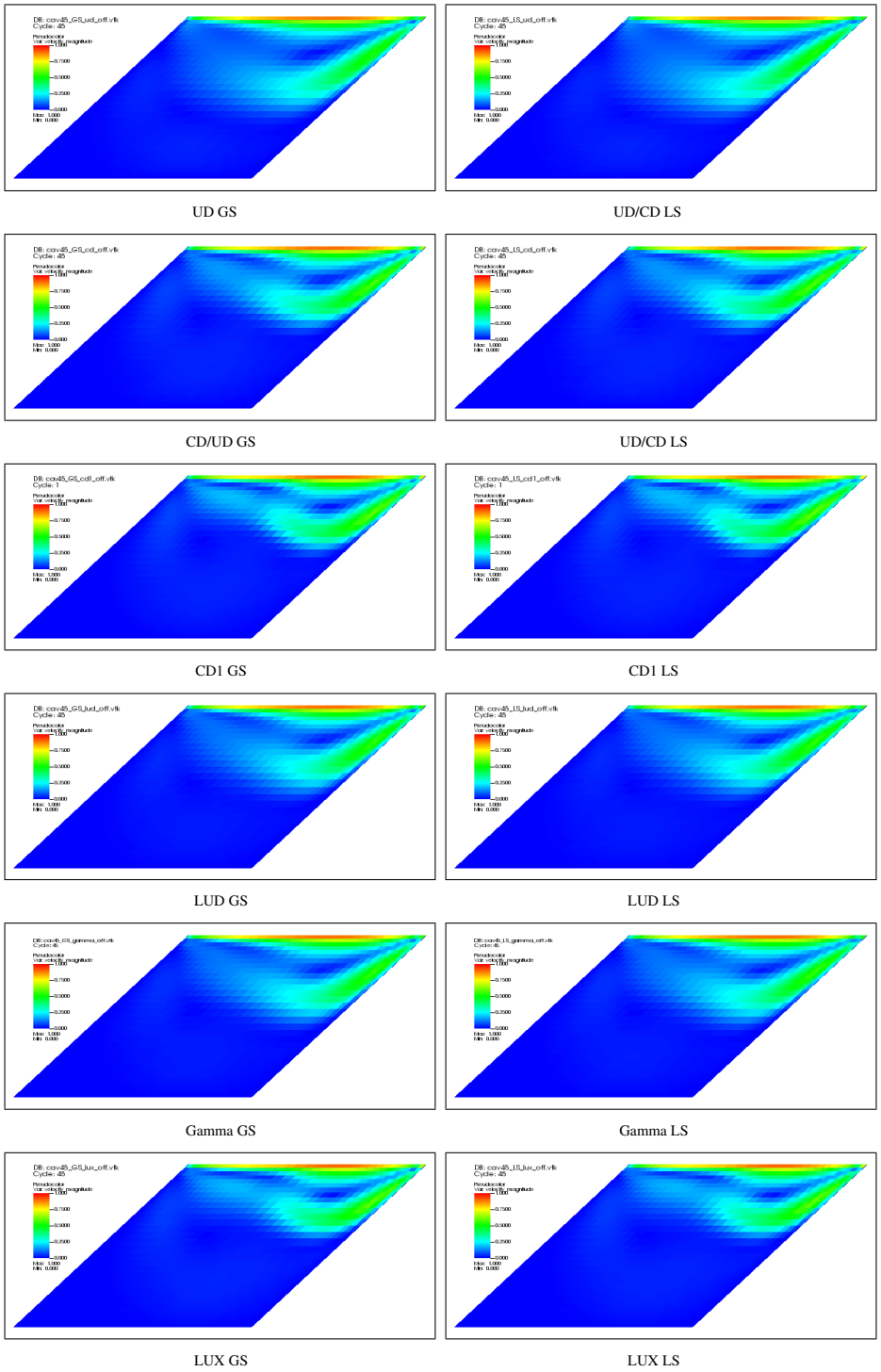


Figure 9.4: cav45 LDC 45 cases with Gauss and Least Squares, no limiters used

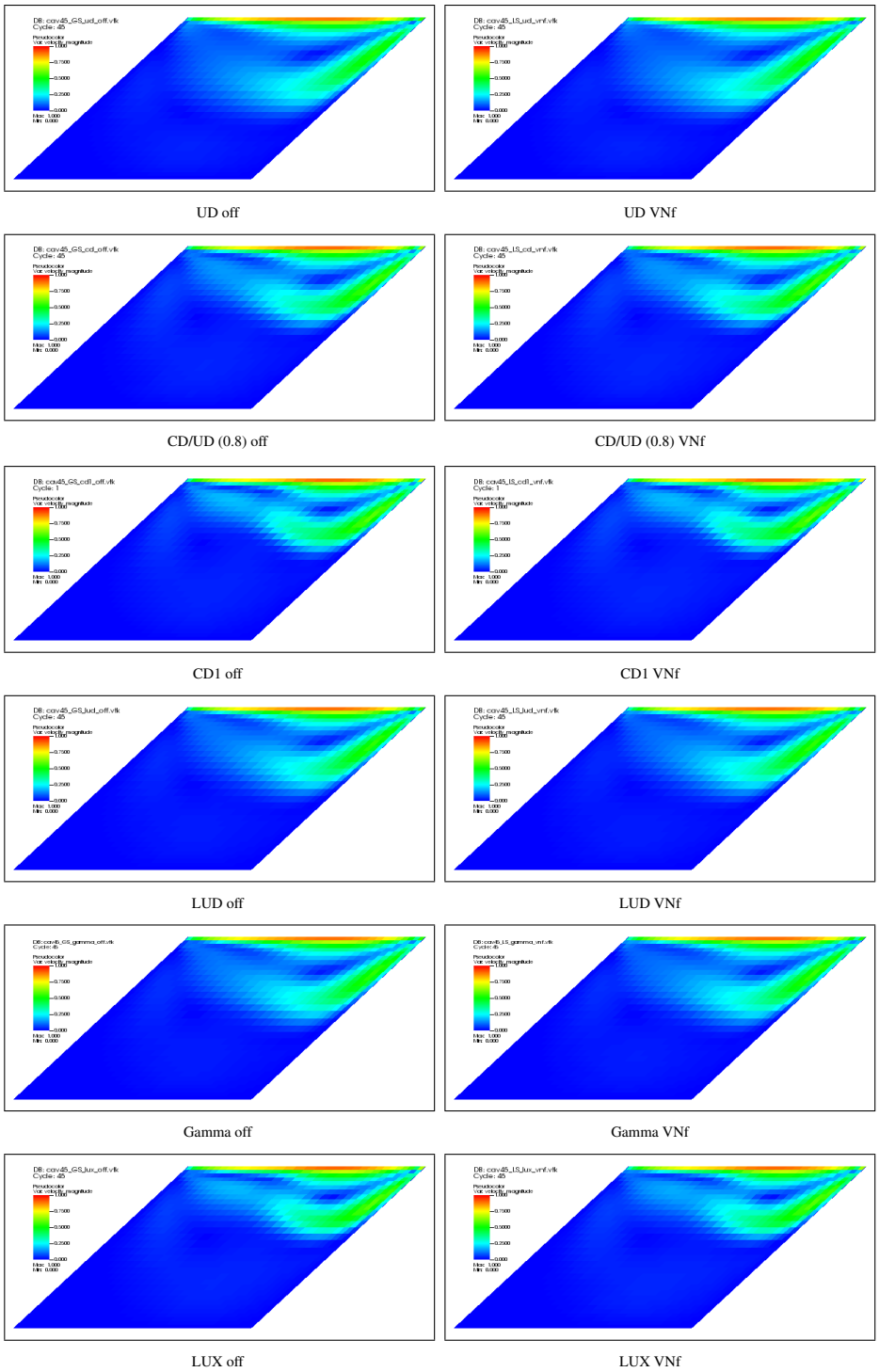


Figure 9.5: cav45 LDC 45 cases with Gauss and face based limiters

9.2 Hexahedra at 20 degrees

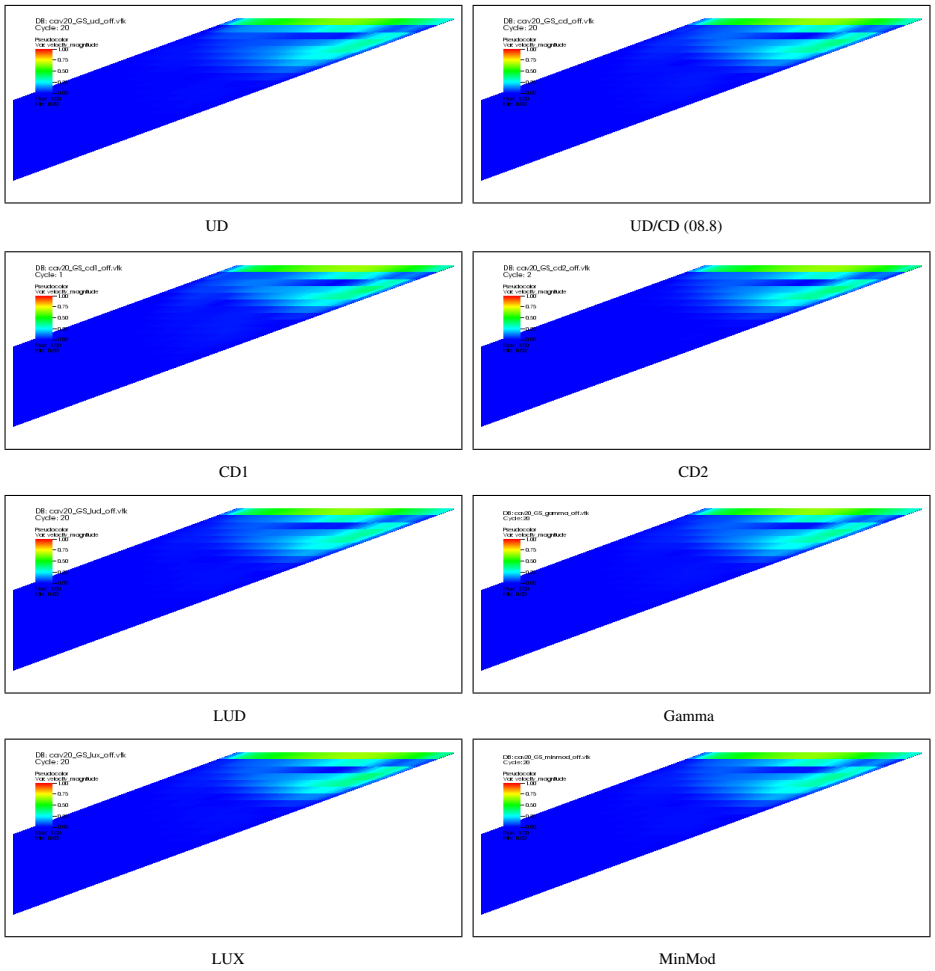
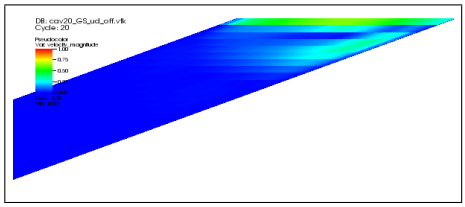
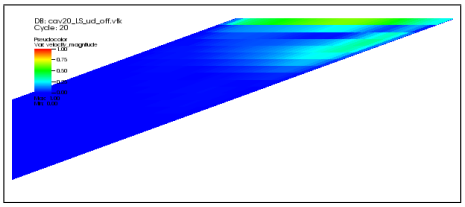


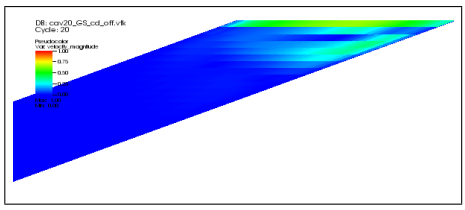
Figure 9.6: cav20 LDC at 20 degrees with Gauss, no limiter



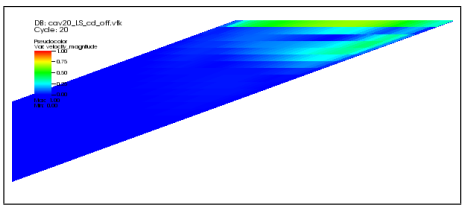
UD GS



UD/CD LS



CD/UD GS



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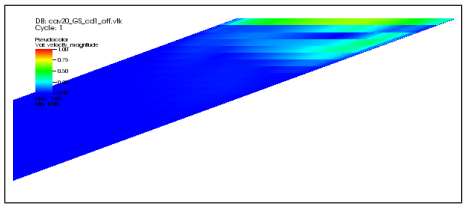
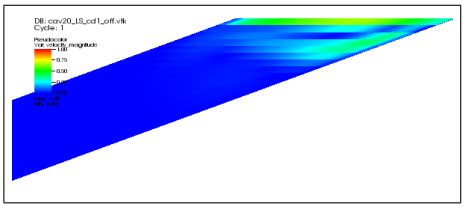
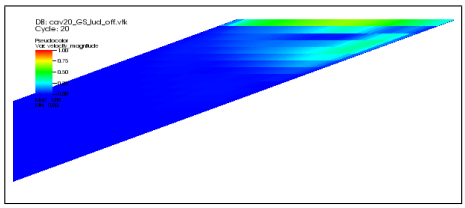


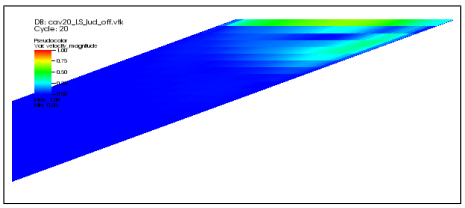
CD1 GS



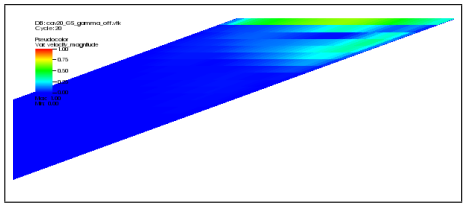
CD1 LS



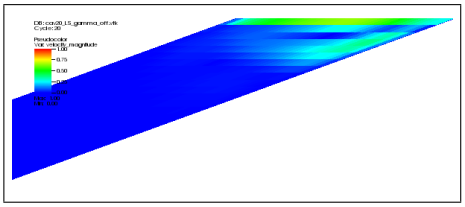
LUD GS



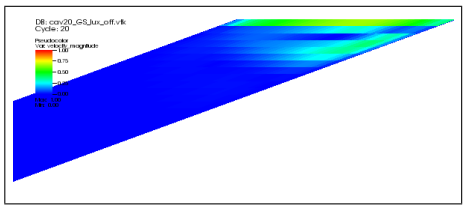
LUD LS



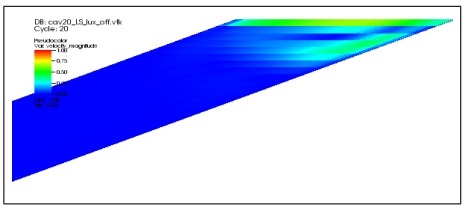
Gamma GS



Gamma LS



LUX GS



LUX LS

Figure 9.7: cav20 LDC 20 cases with Gauss and Least Squares, no limiters used

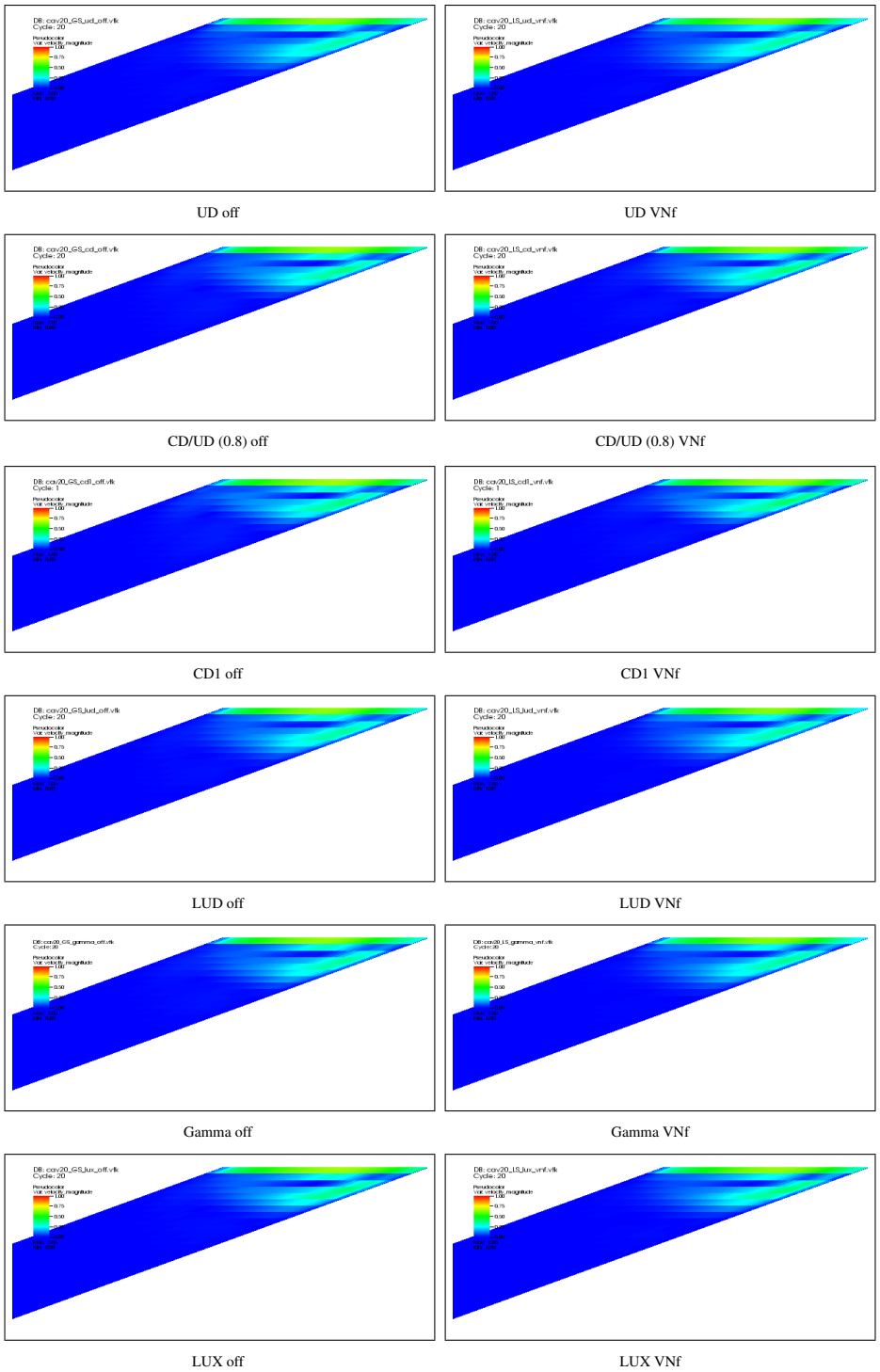


Figure 9.8: cav20 LDC 20 cases with Gauss and face based limiters

10 W1/W2 Wedge cell tests

10.1 Step cases

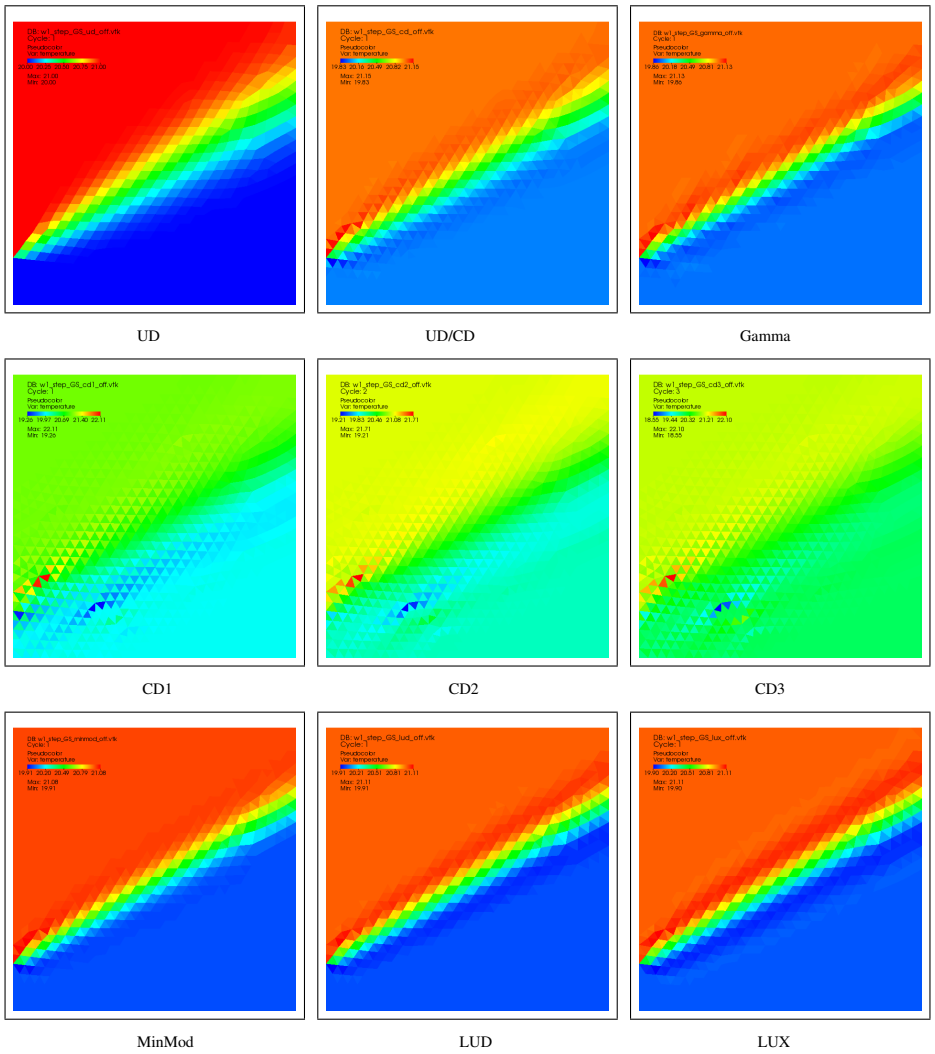


Figure 10.1: Standard Leonard step cases with Gauss, no limiter

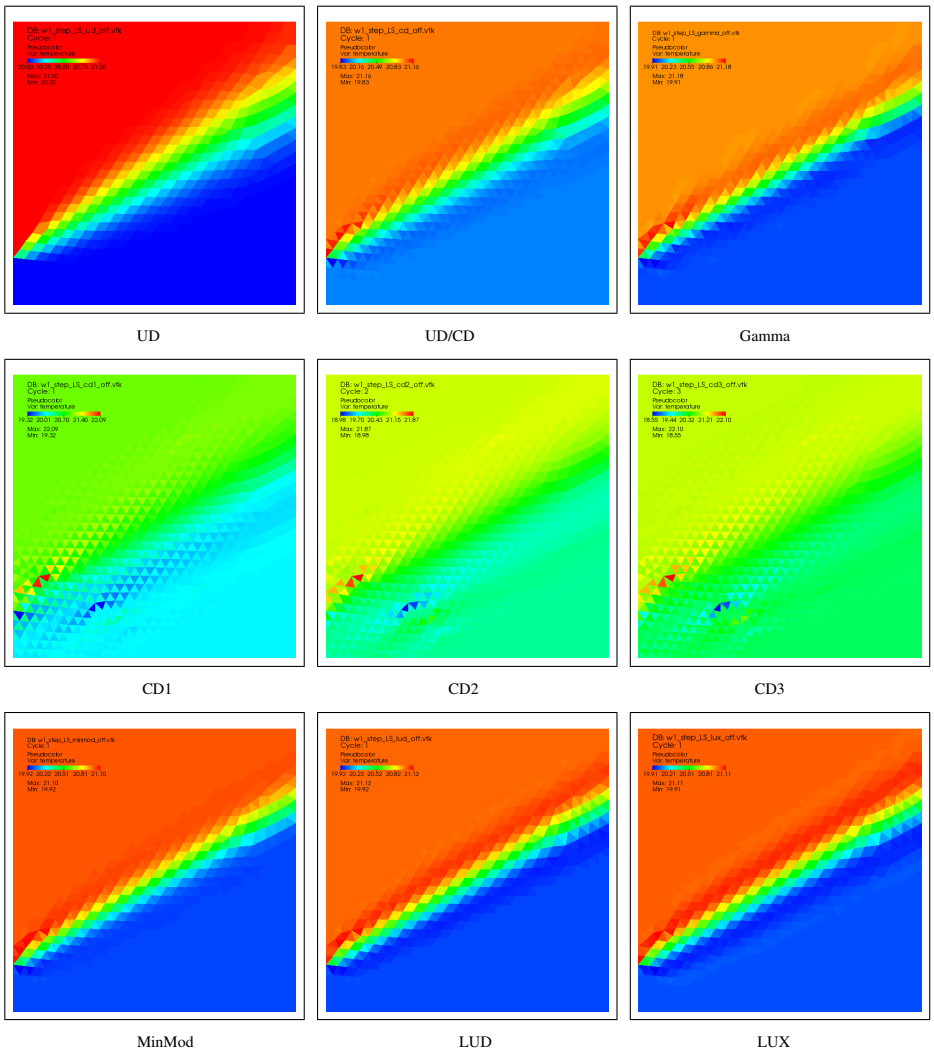


Figure 10.2: Standard Leonard step cases with Least Squares, no limiter

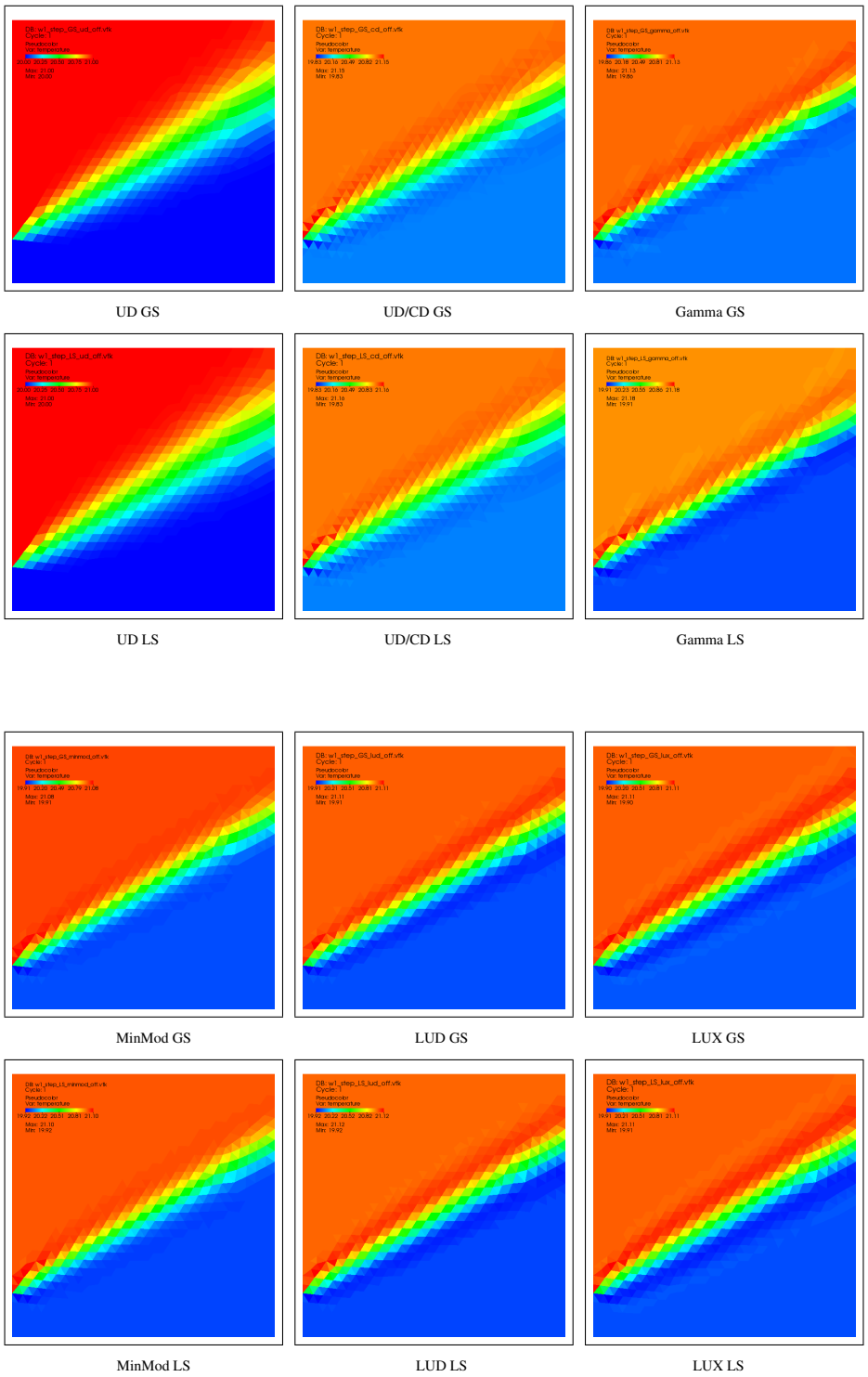


Figure 10.3: Standard Leonard step cases with Gauss (GS) and Least Squares (LS), no limiters used

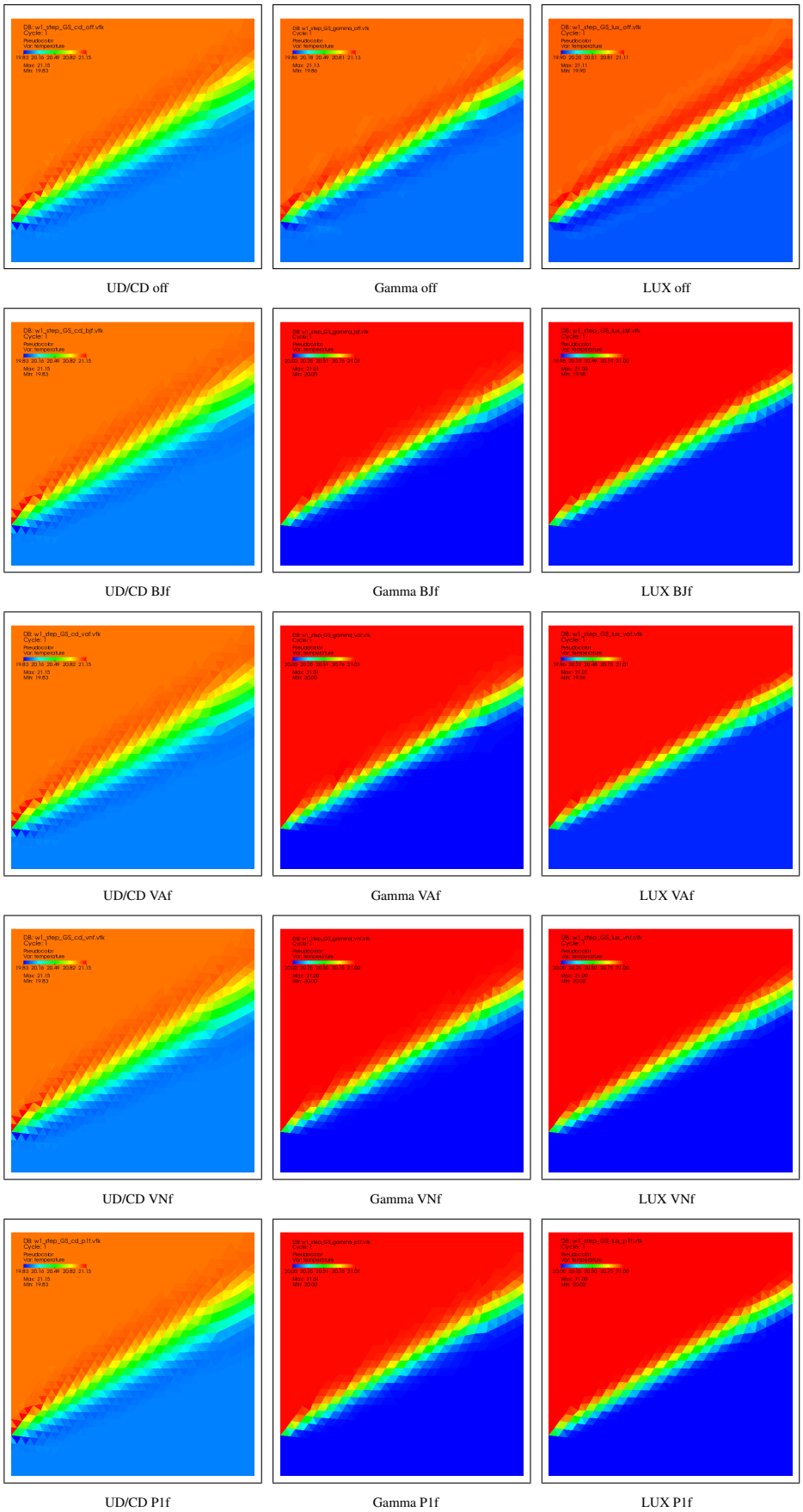


Figure 10.4: Leonard step cases with Gauss and face based limiters

10.2 Sin2 cases

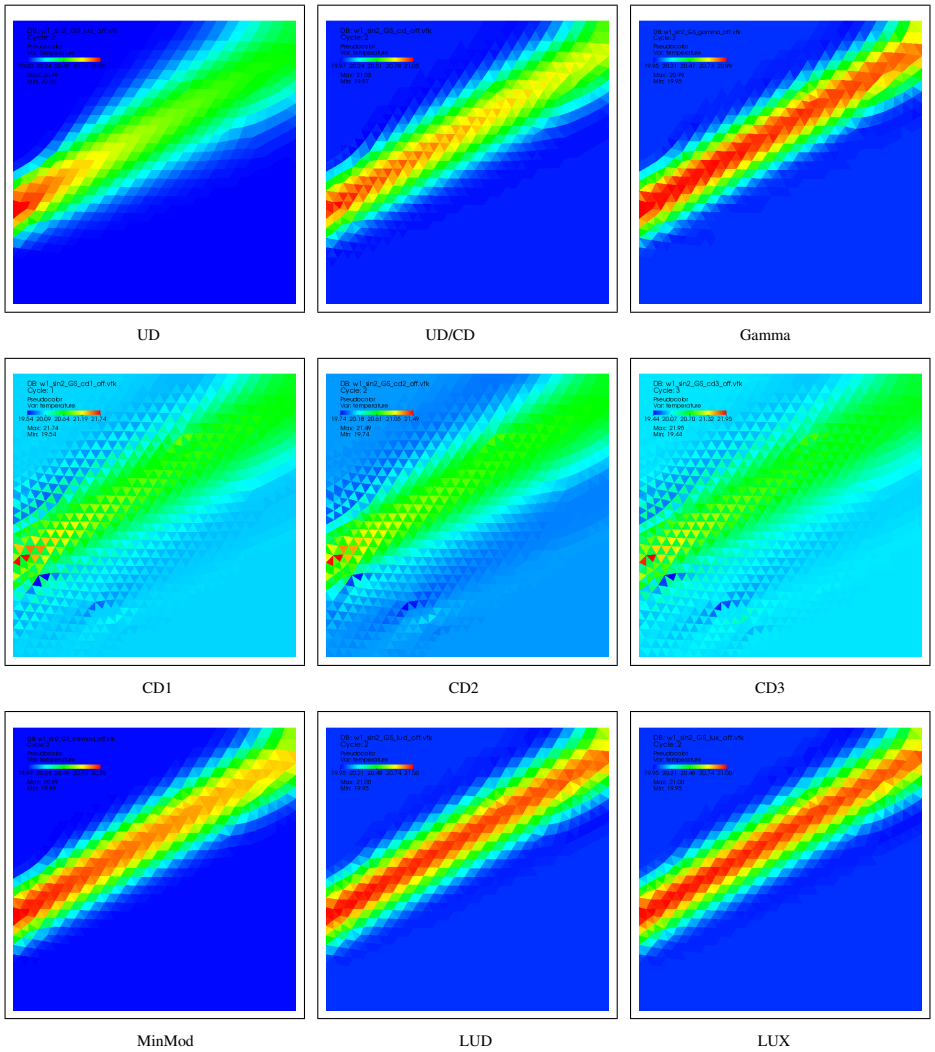


Figure 10.5: Standard Leonard sin2 cases with Gauss, no limiter

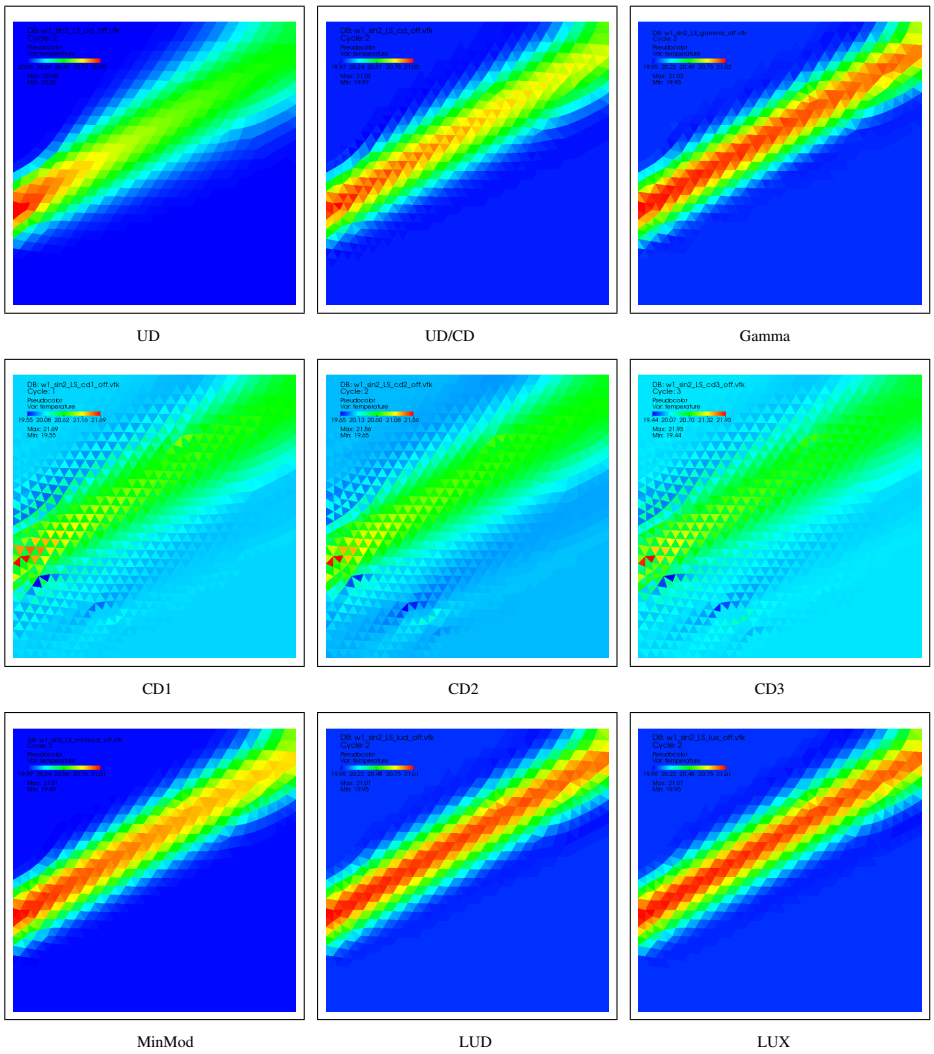


Figure 10.6: Standard Leonard sin2 cases with Least Squares, no limiter

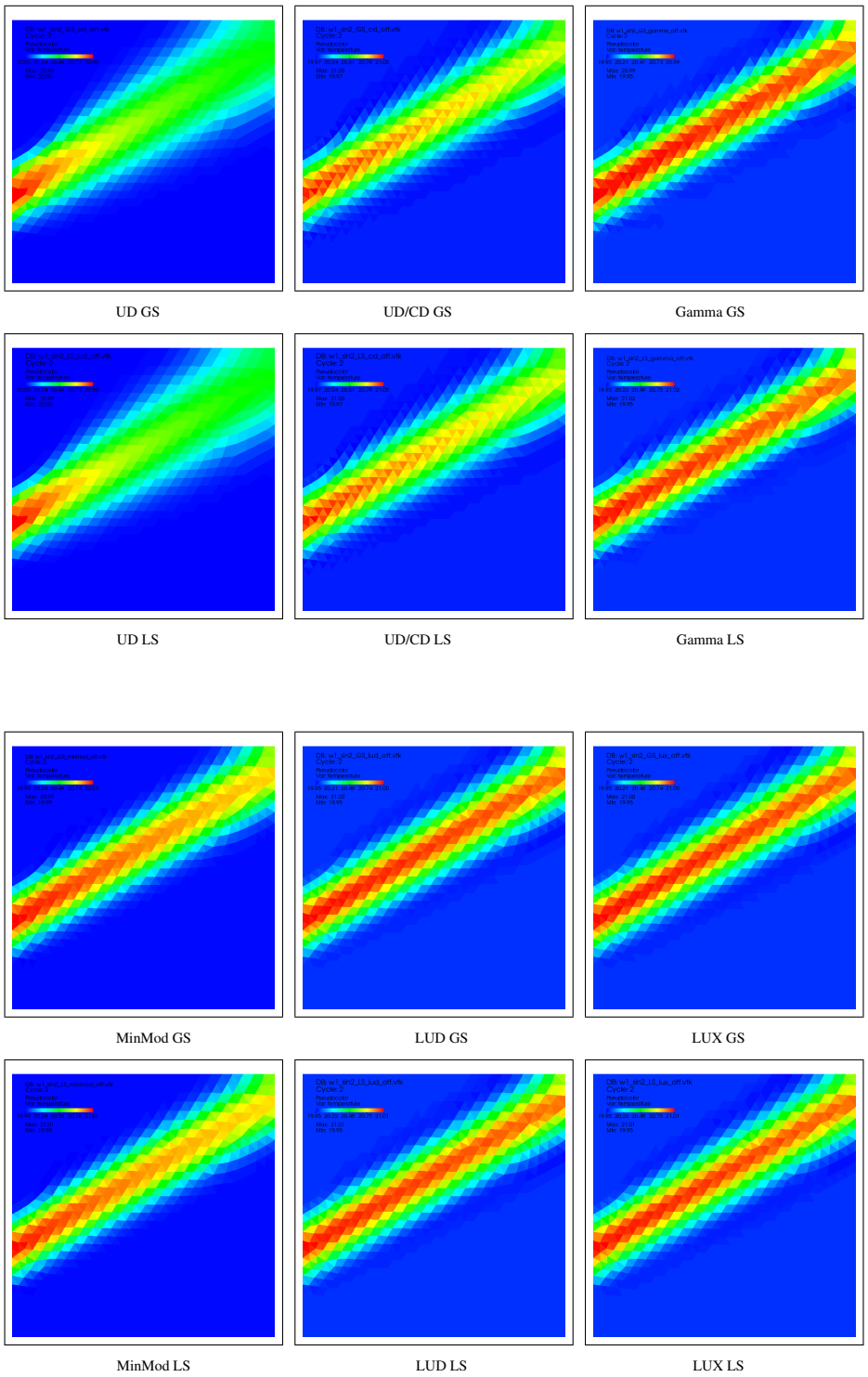


Figure 10.7: Standard Leonard sin2 cases with Gauss (GS) and Least Squares (LS), no limiters used

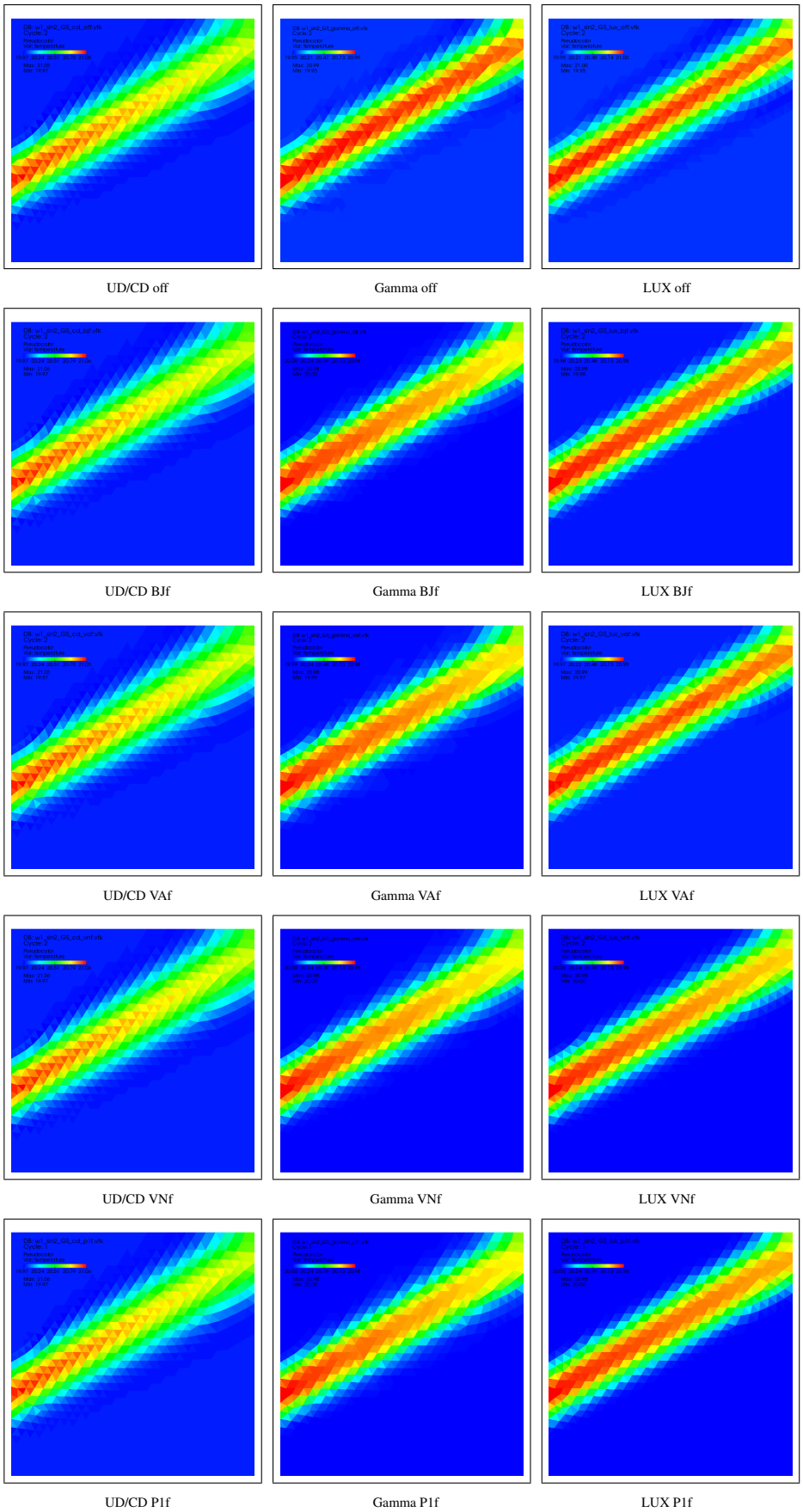


Figure 10.8: Leonard sin2 cases with Gauss and face based limiters

10.3 Semi ellipse cases

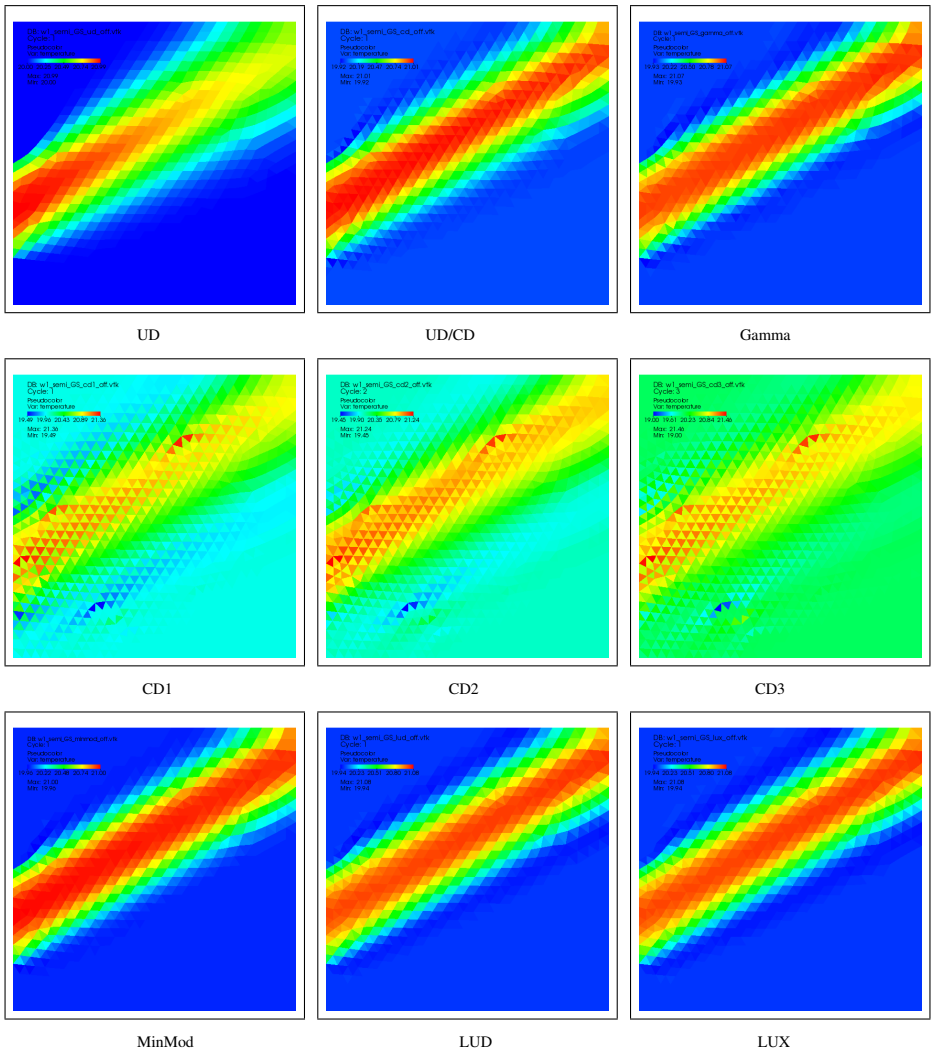


Figure 10.9: Standard Leonard semi cases with Gauss, no limiter

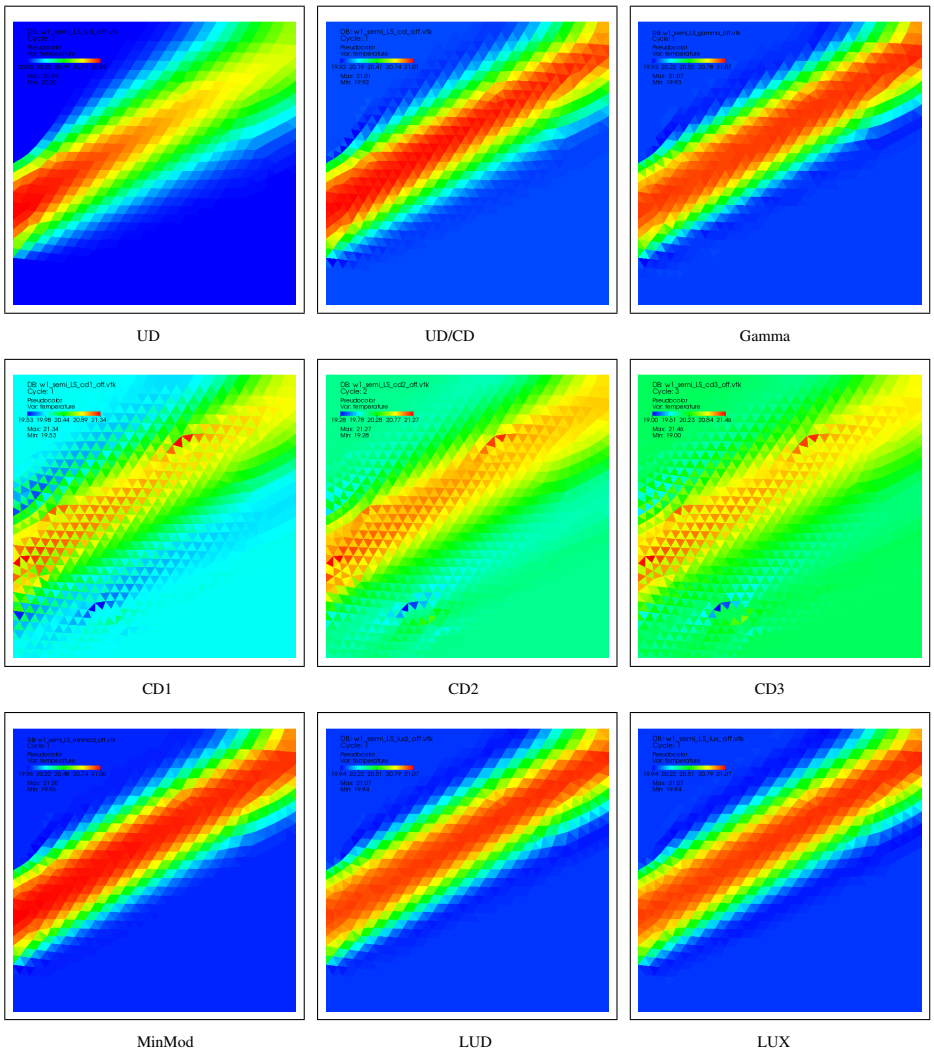


Figure 10.10: Standard Leonard semi cases with Least Squares, no limiter

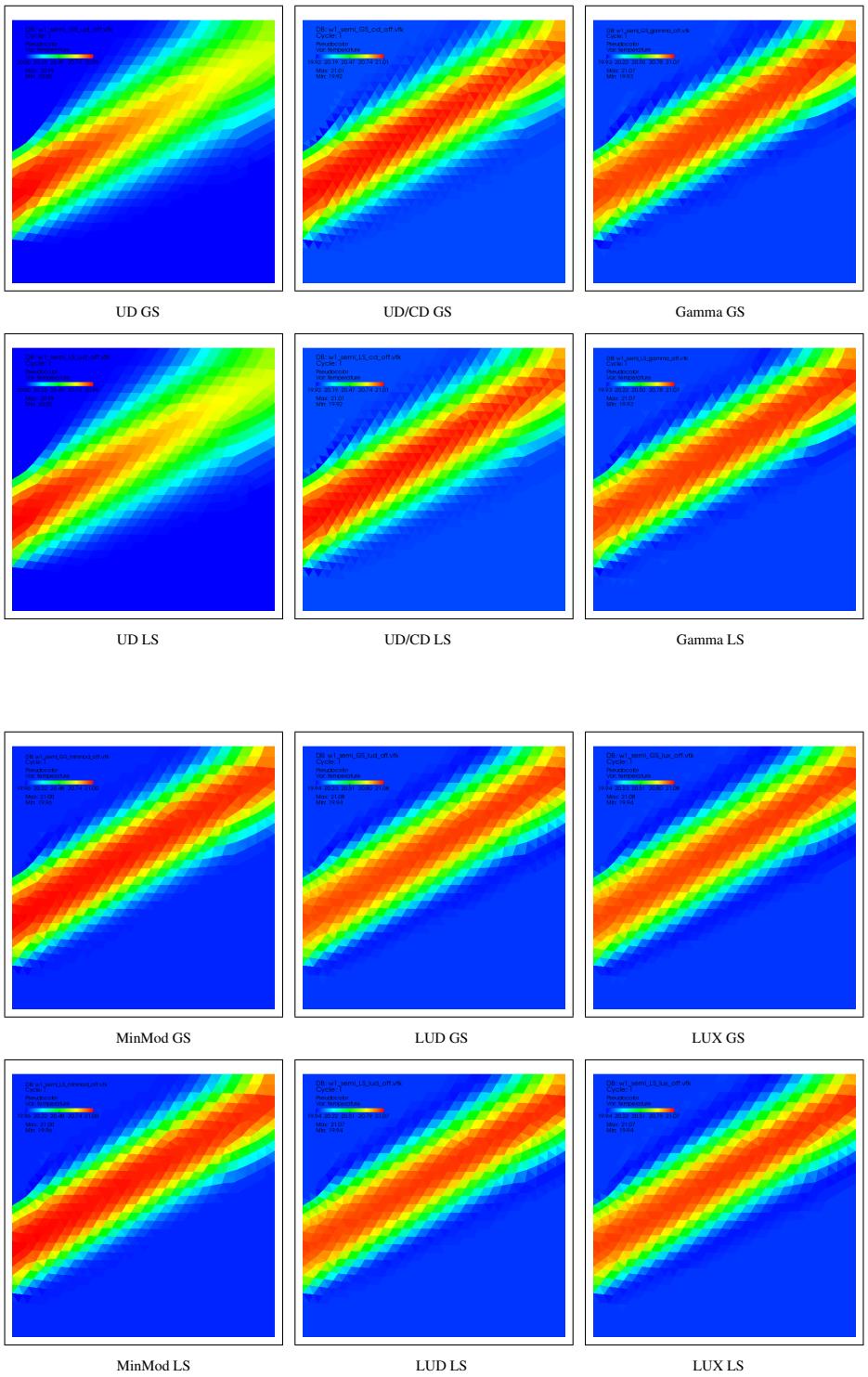


Figure 10.11: Standard Leonard semi cases with Gauss (GS) and Least Squares (LS), no limiters used

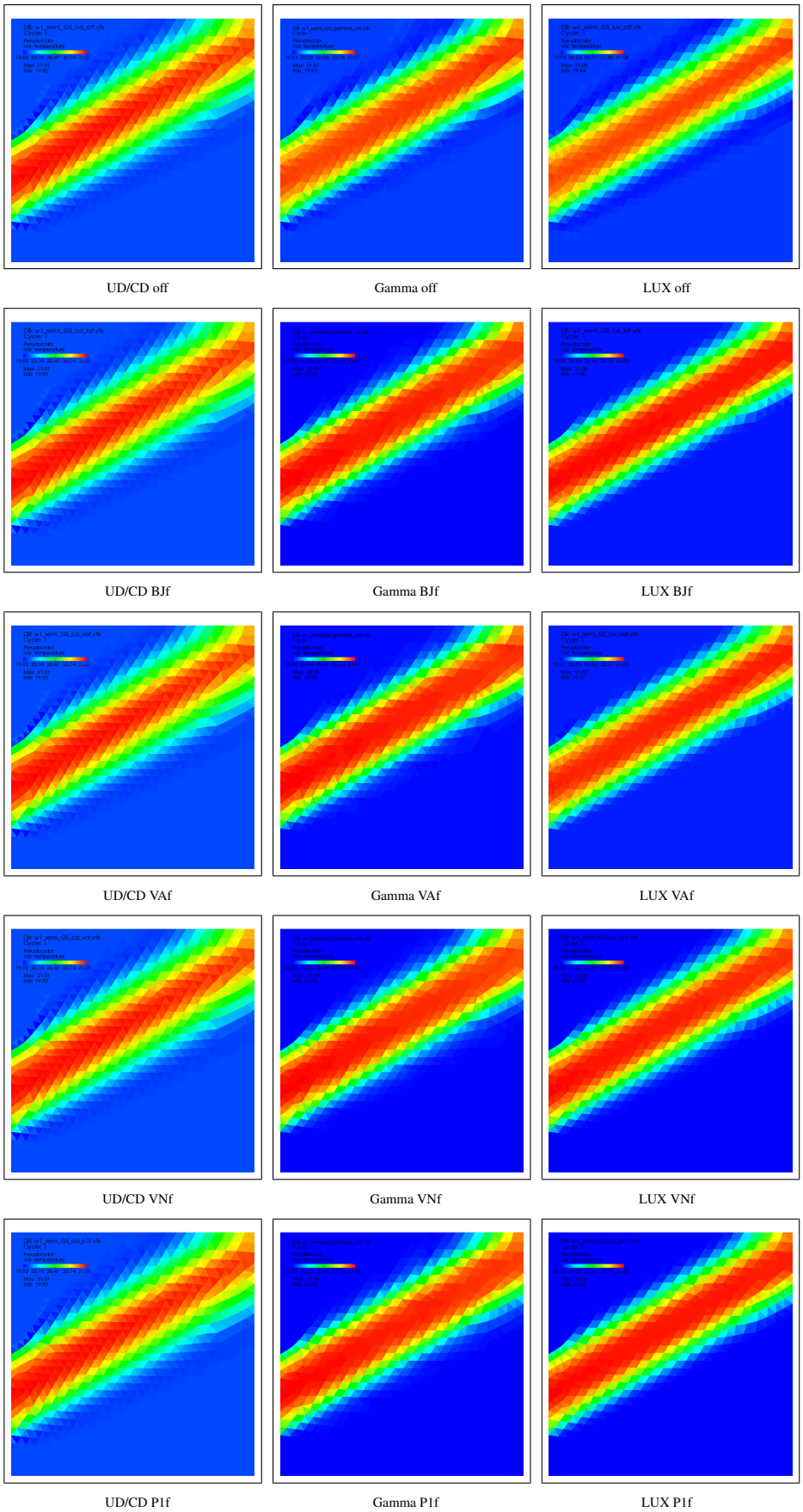


Figure 10.12: Leonard semi cases with Gauss and face based limiters

10.4 Lid driven cavity

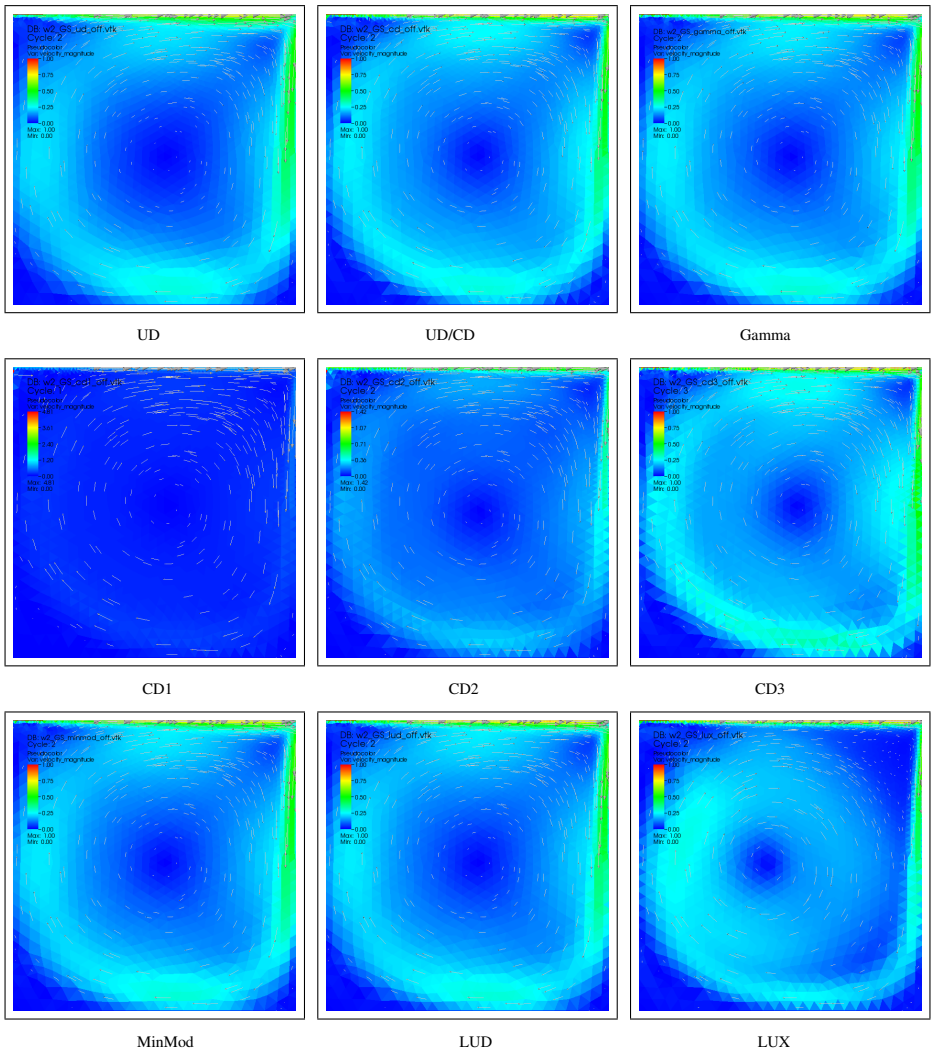


Figure 10.13: W2 LDC wedge cases with Gauss, no limiter

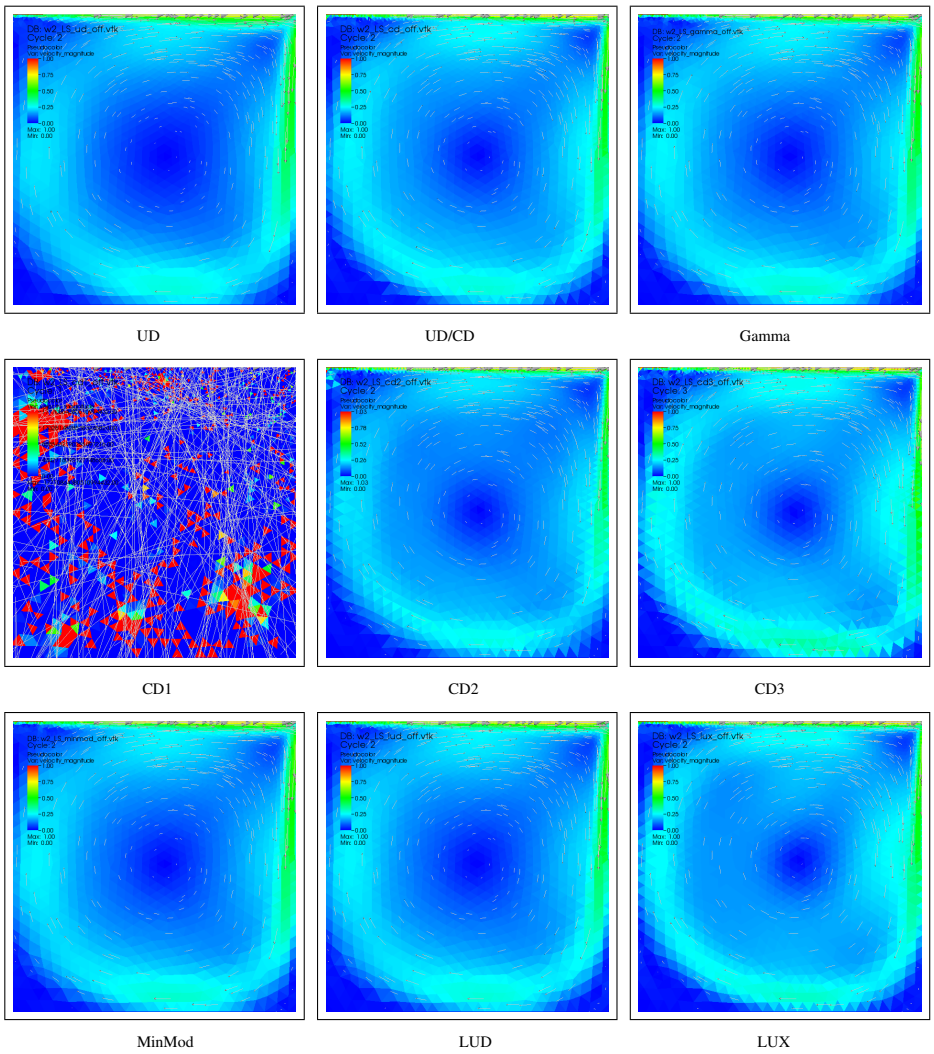


Figure 10.14: W2 LDC wedge cases with Least Squares, no limiter

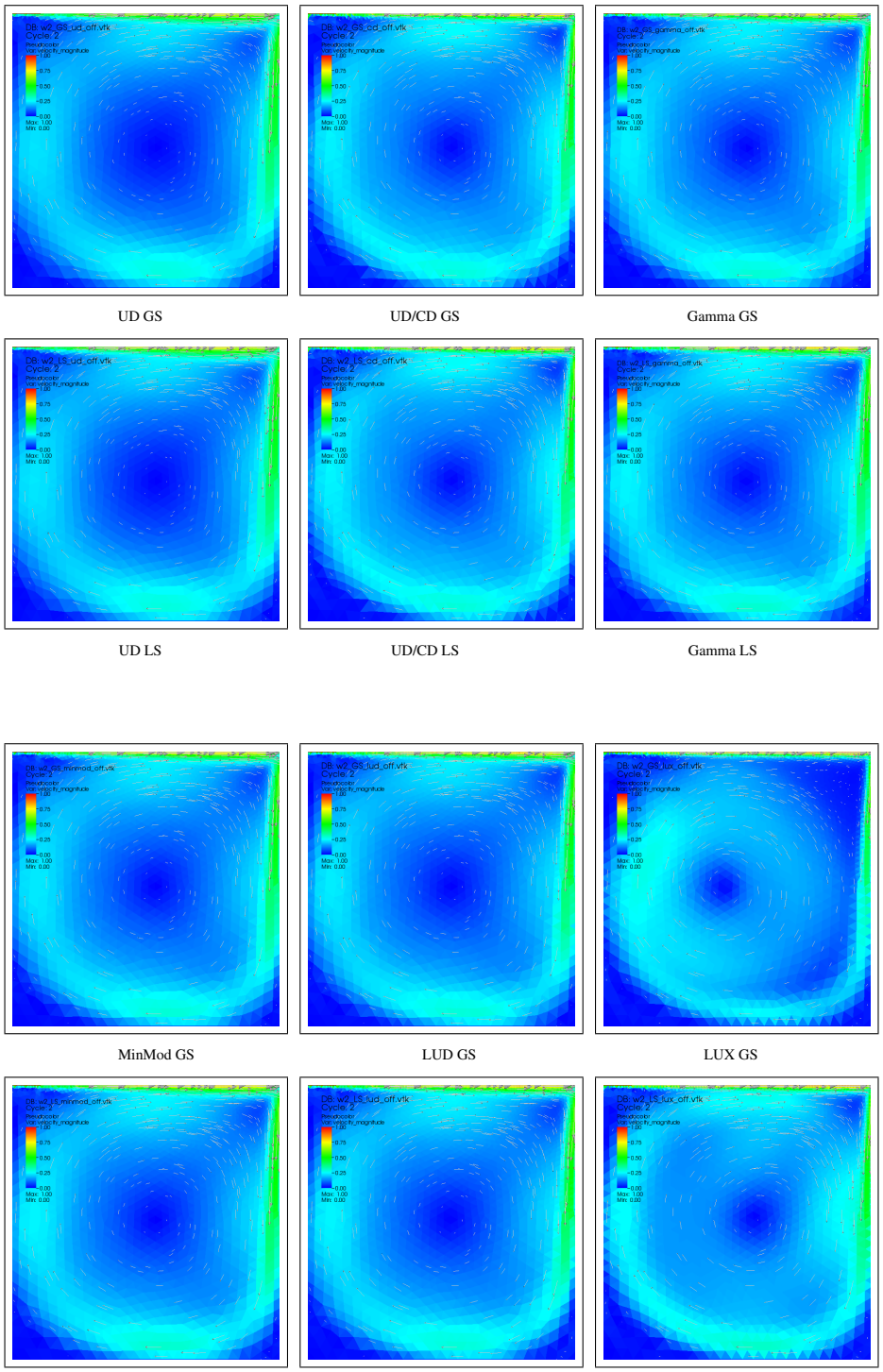


Figure 10.15: W2 LDC wedge cases with Gauss and Least Squares, no limiters used

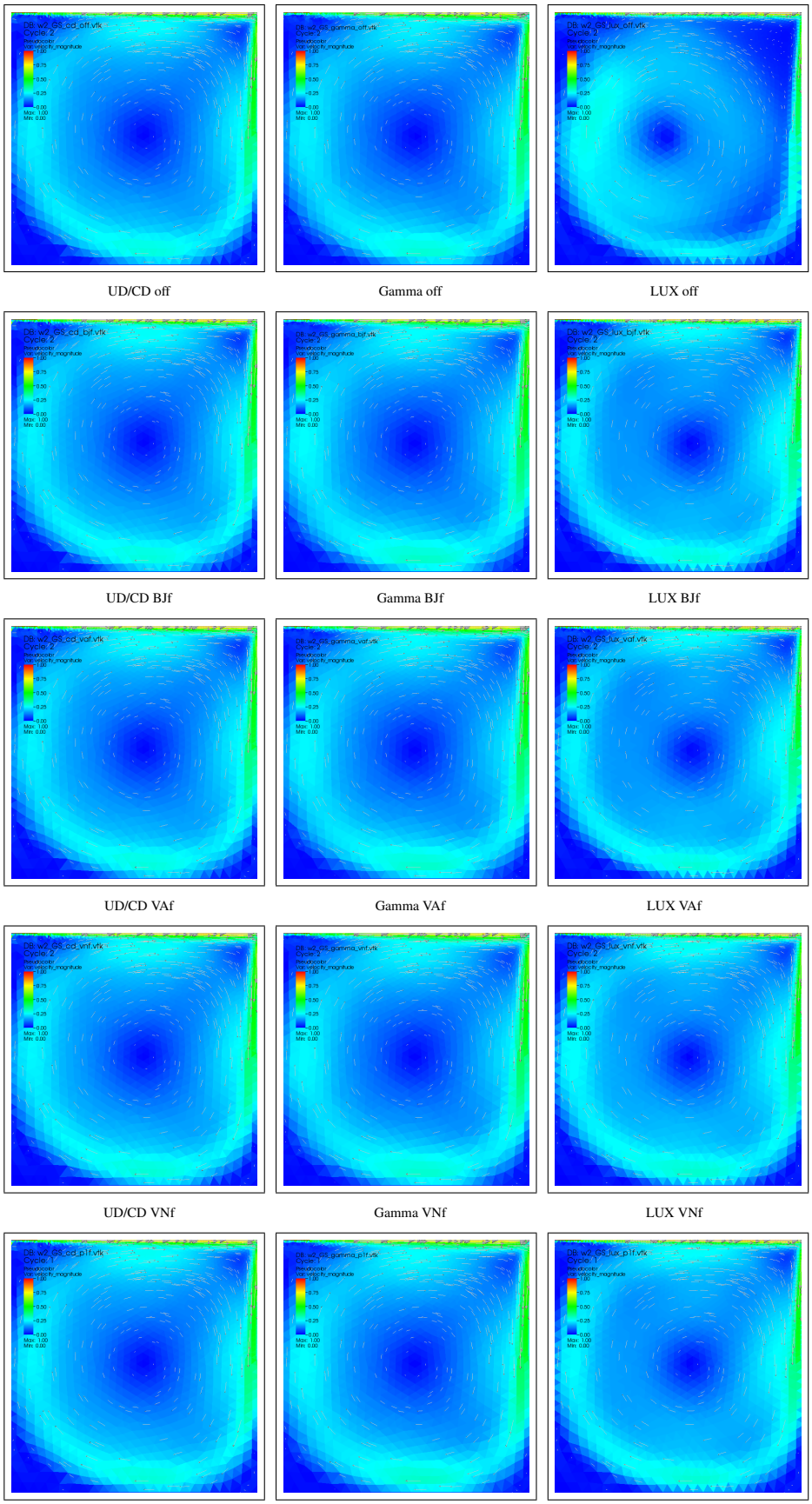


Figure 10.16: W2 LDC wedge cases with Gauss and face based limiters

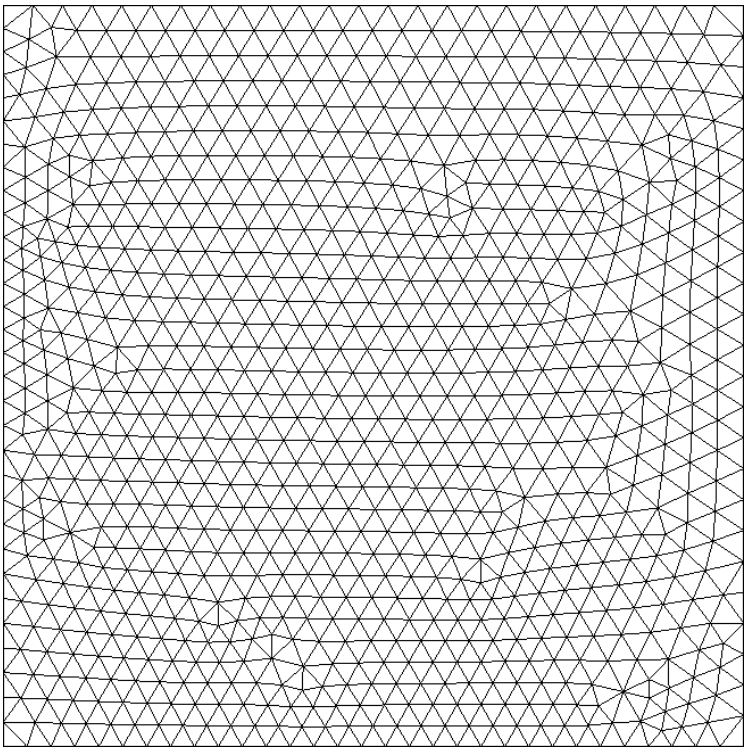


Figure 10.17: Mesh model w1

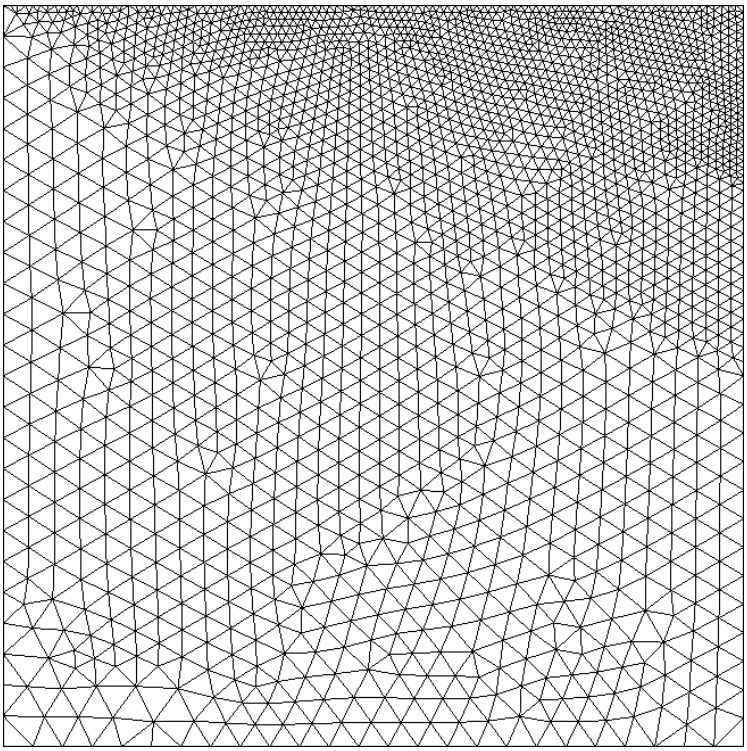


Figure 10.18: Mesh model w2

11 M3/M5 Tetrahedral cell tests

11.1 Lid driven cavity m3

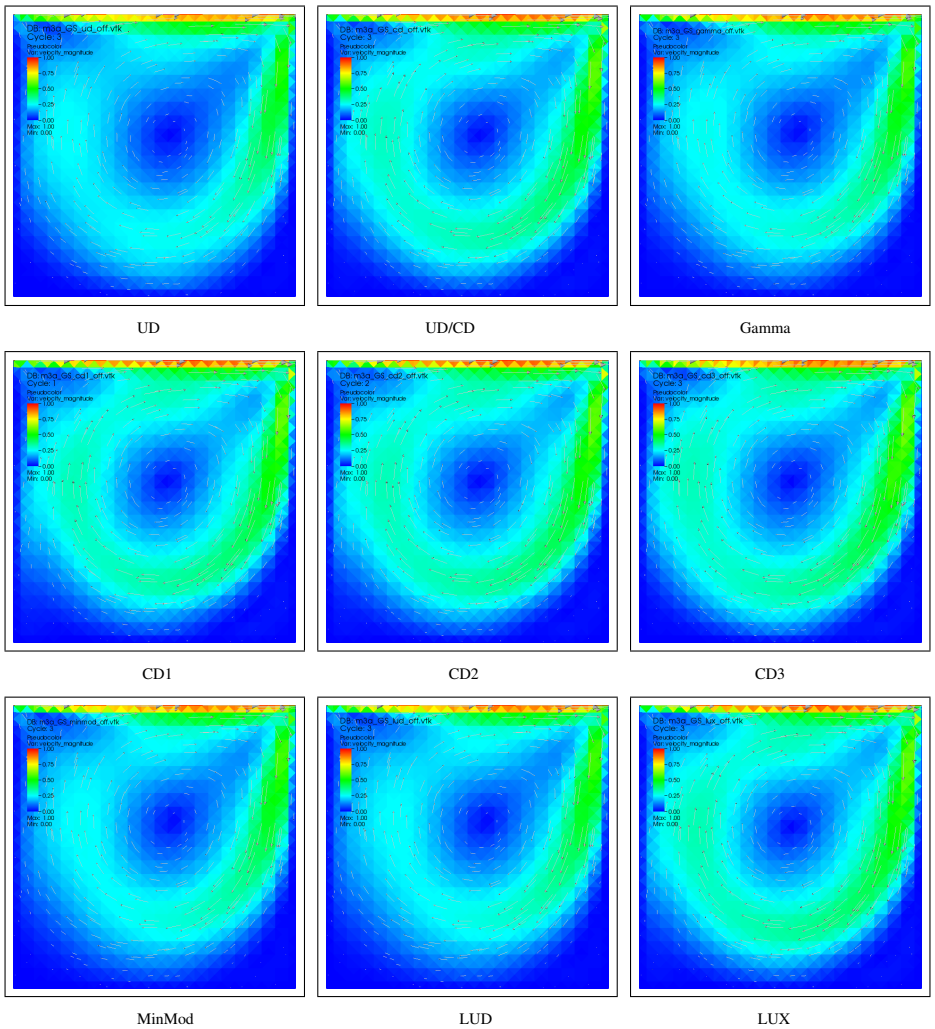


Figure 11.1: m3a LDC tet cases with Gauss, no limiter

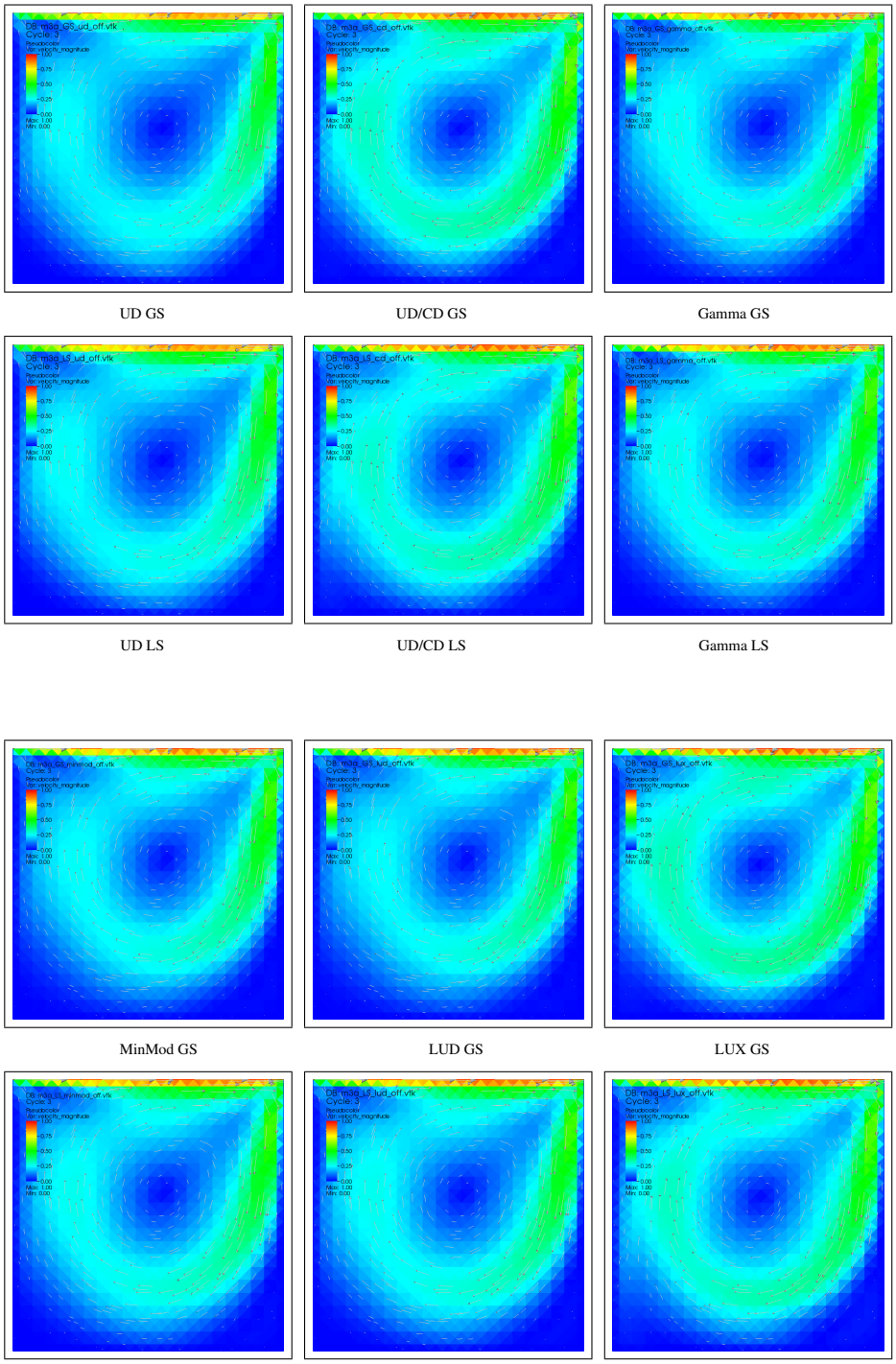


Figure 11.2: m3a LDC tet cases with Gauss and Least Squares, no limiters used

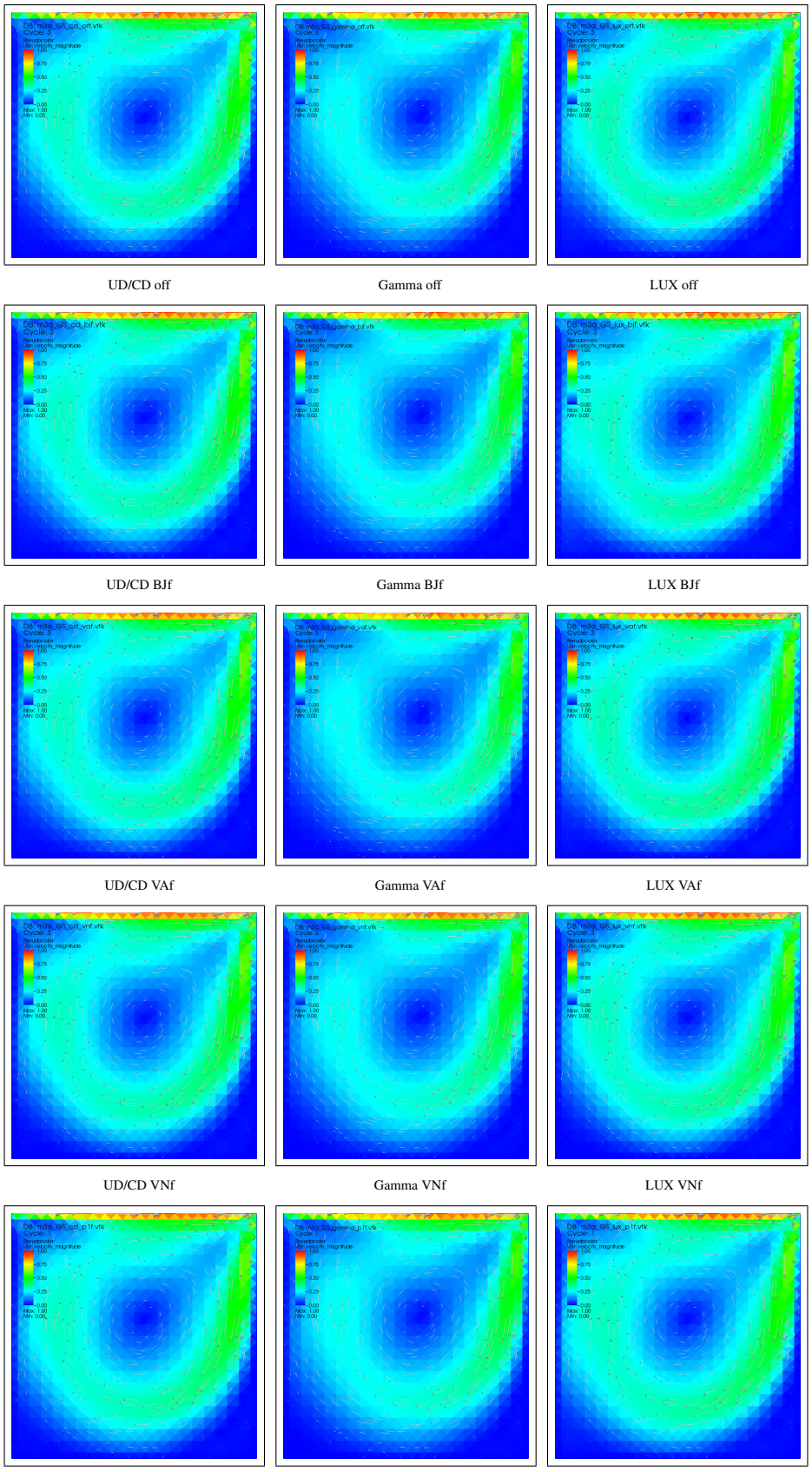


Figure 11.3: m3a LDC tet cases with Gauss and face based limiters

11.2 Plain flow from left to right

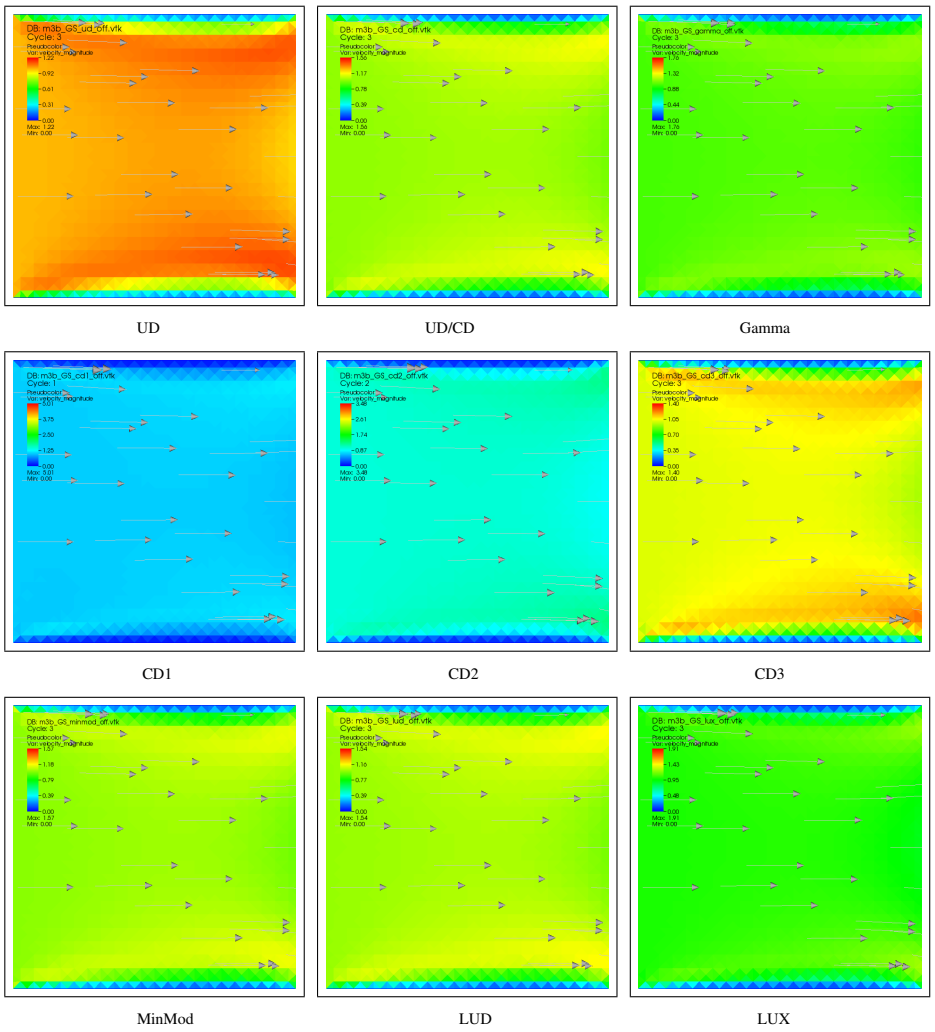


Figure 11.4: m3b plain flow tet cases with Gauss, no limiter

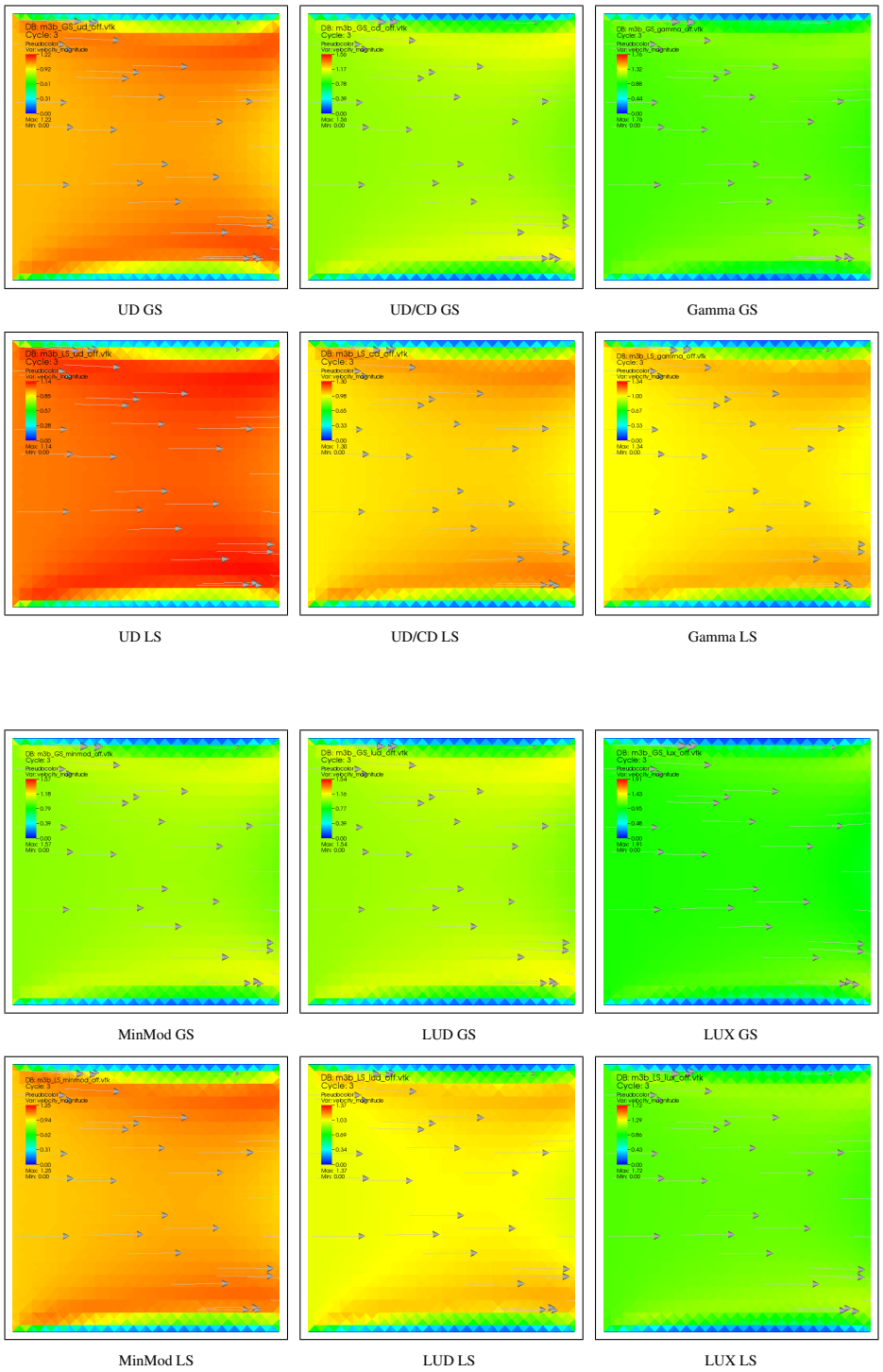


Figure 11.5: m3b plain flow tet cases with Gauss and Least Squares, no limiters used

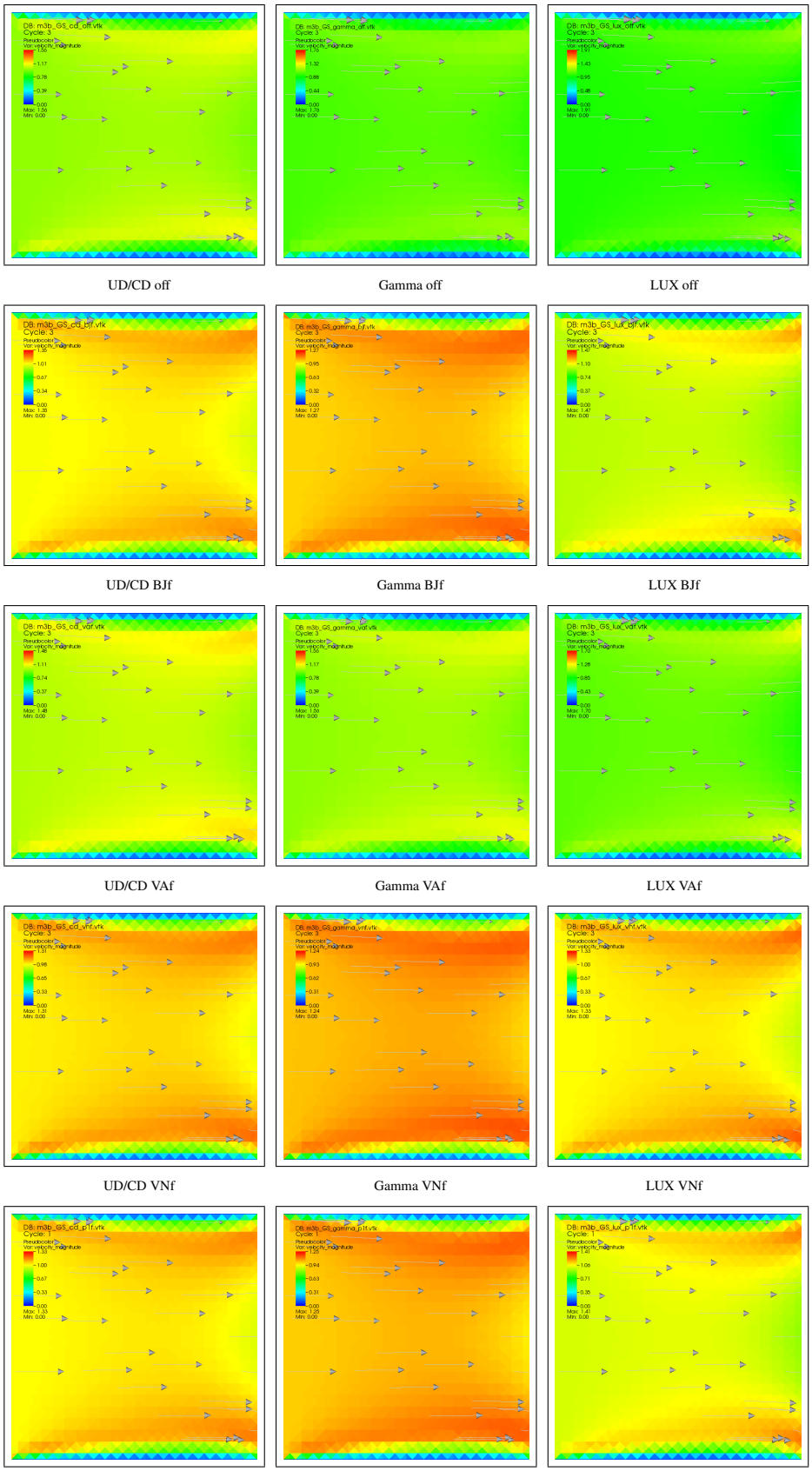


Figure 11.6: m3b plain flow cases with Gauss and face based limiters

11.3 Stagnation flow from top

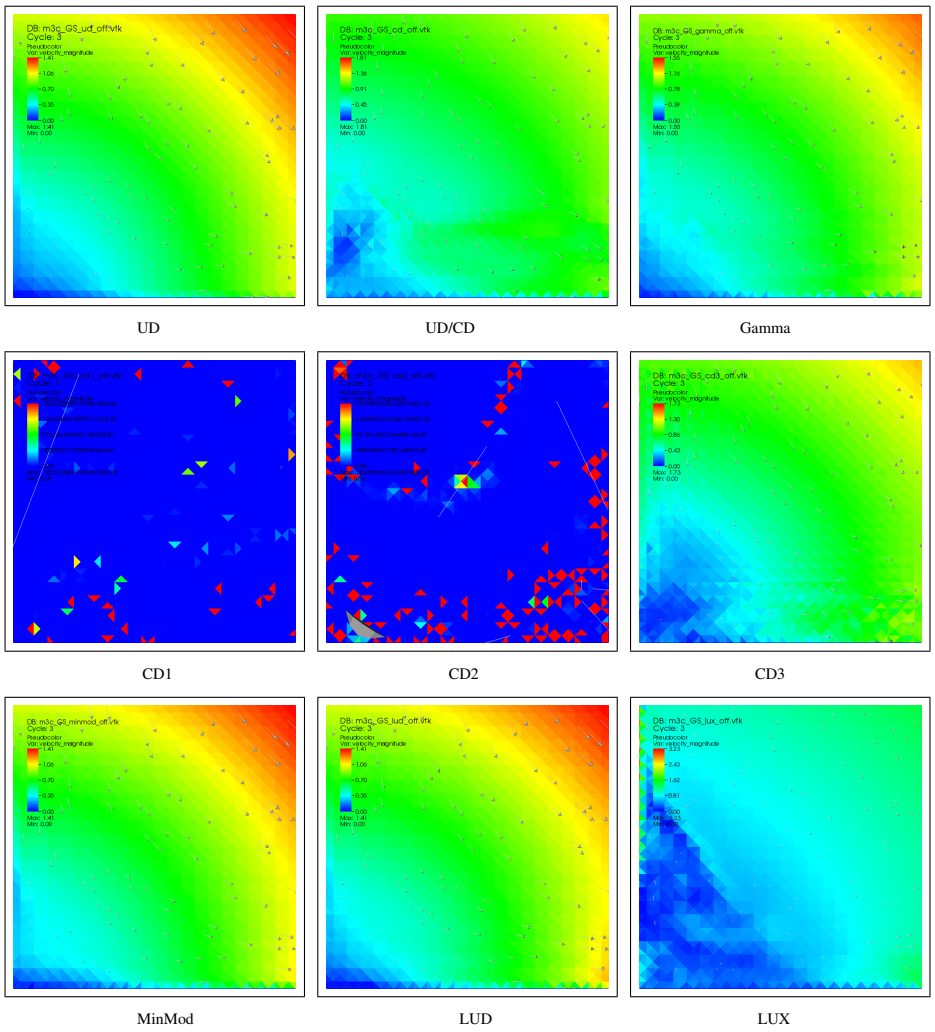


Figure 11.7: m3c stagnation flow tet cases with Gauss, no limiter

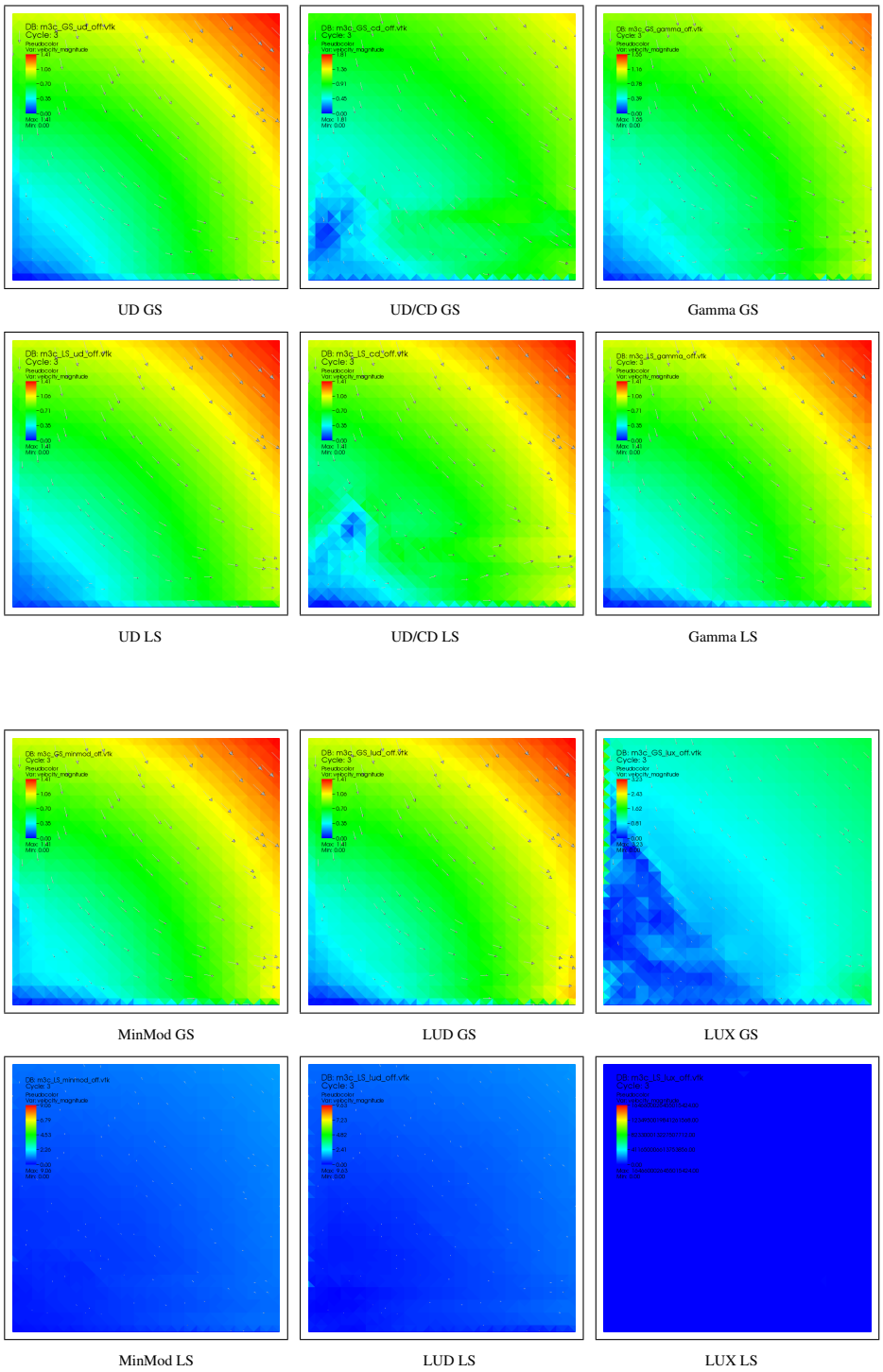


Figure 11.8: m3c stagnation flow tet cases with Gauss and Least Squares, no limiters used

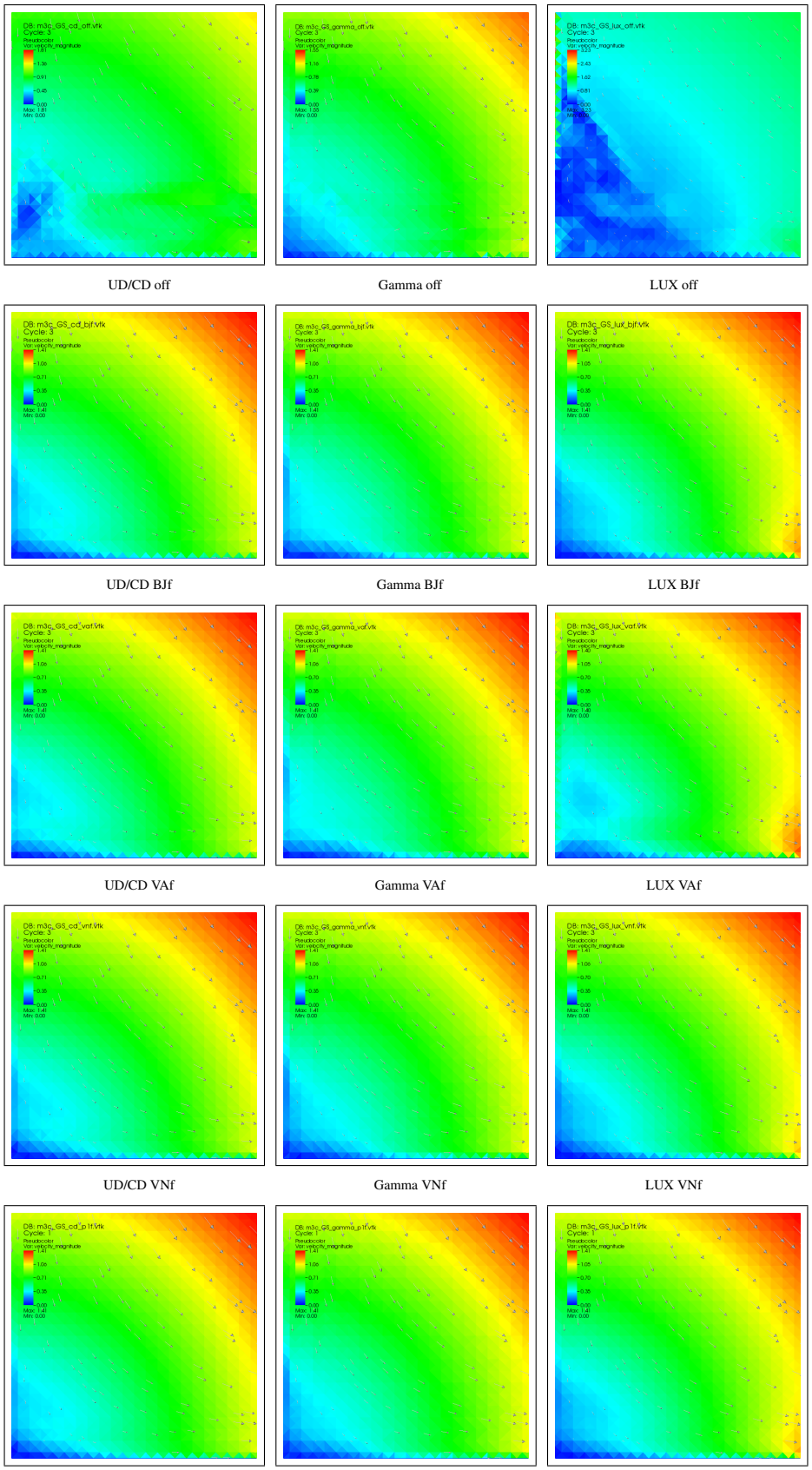


Figure 11.9: m3c stagnation flow cases with Gauss and face based limiters

11.4 Lid driven cavity m5

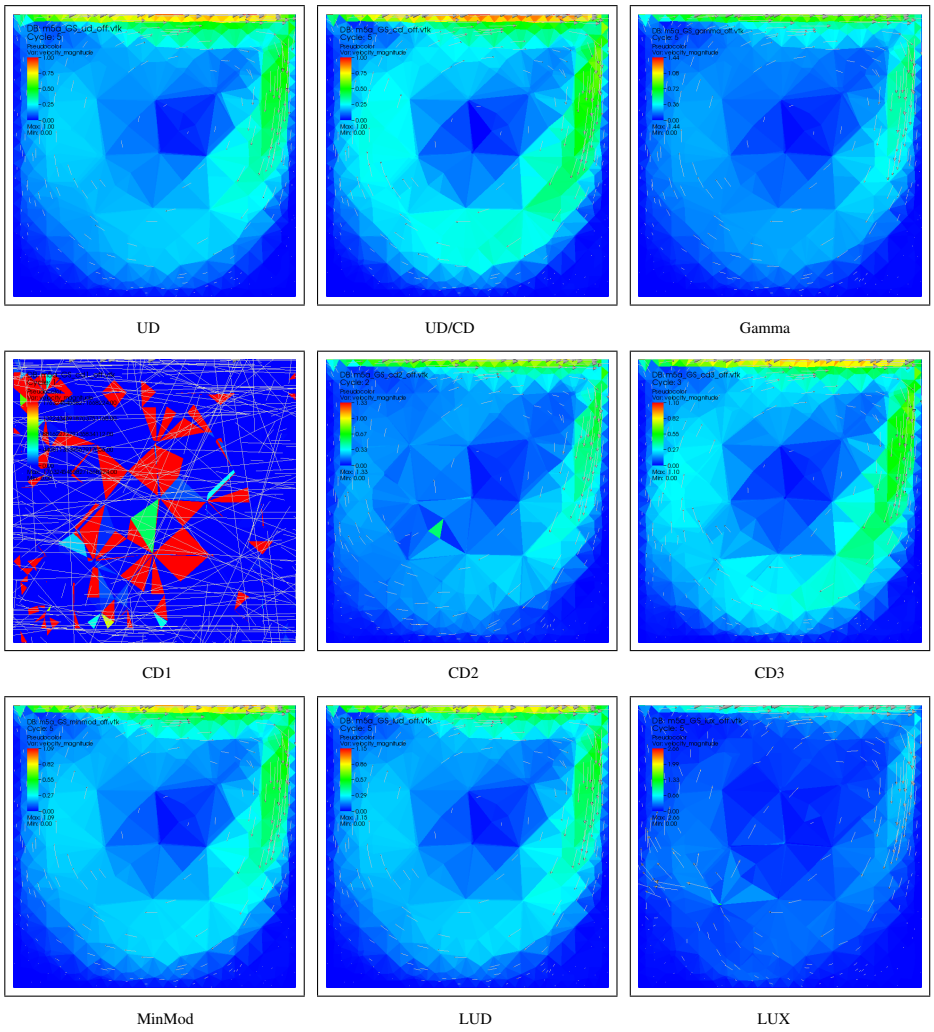


Figure 11.10: m5a LDC tet cases with Gauss, no limiter

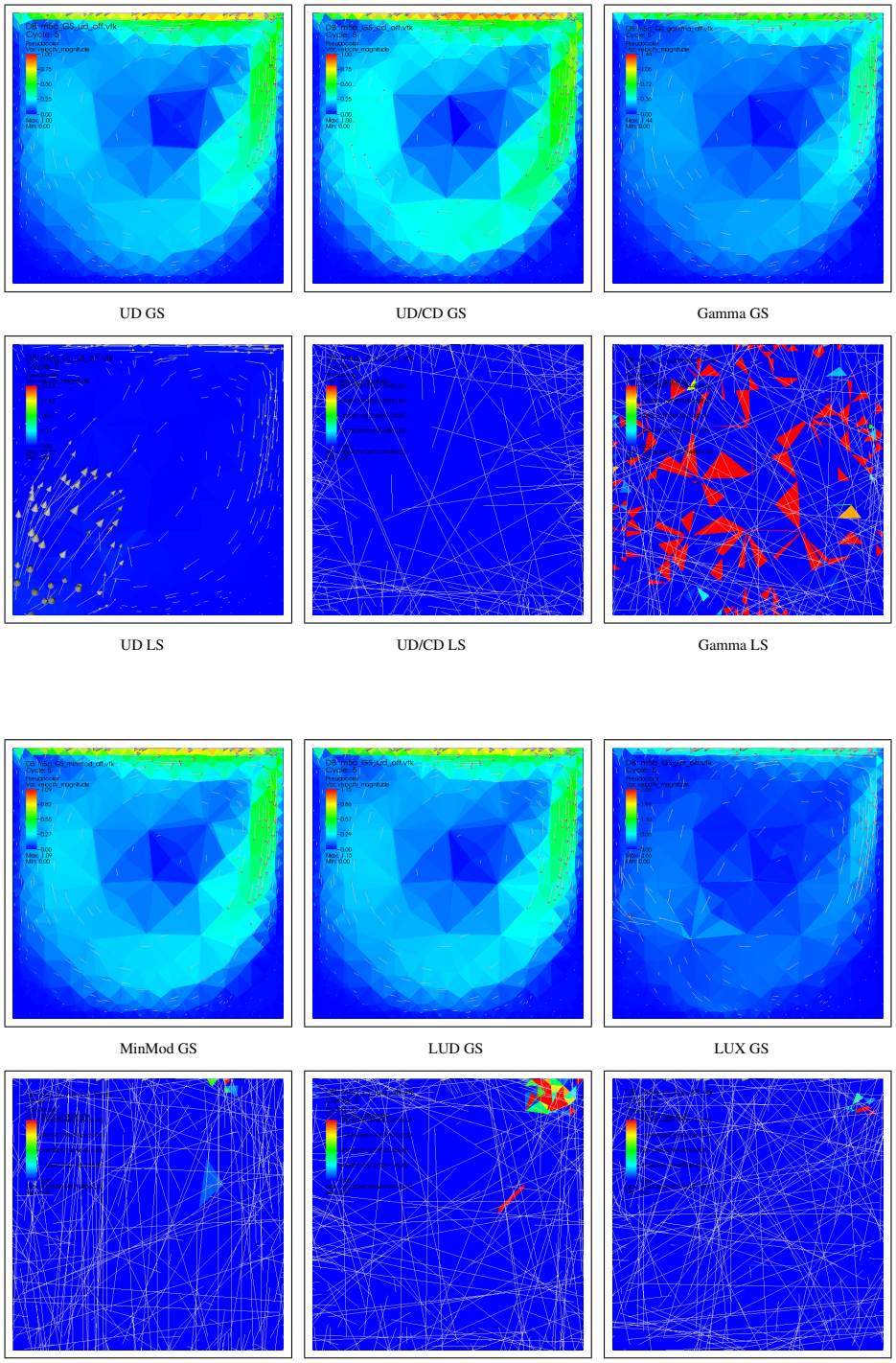


Figure 11.11: m5a LDC tet cases with Gauss and Least Squares, no limiters used

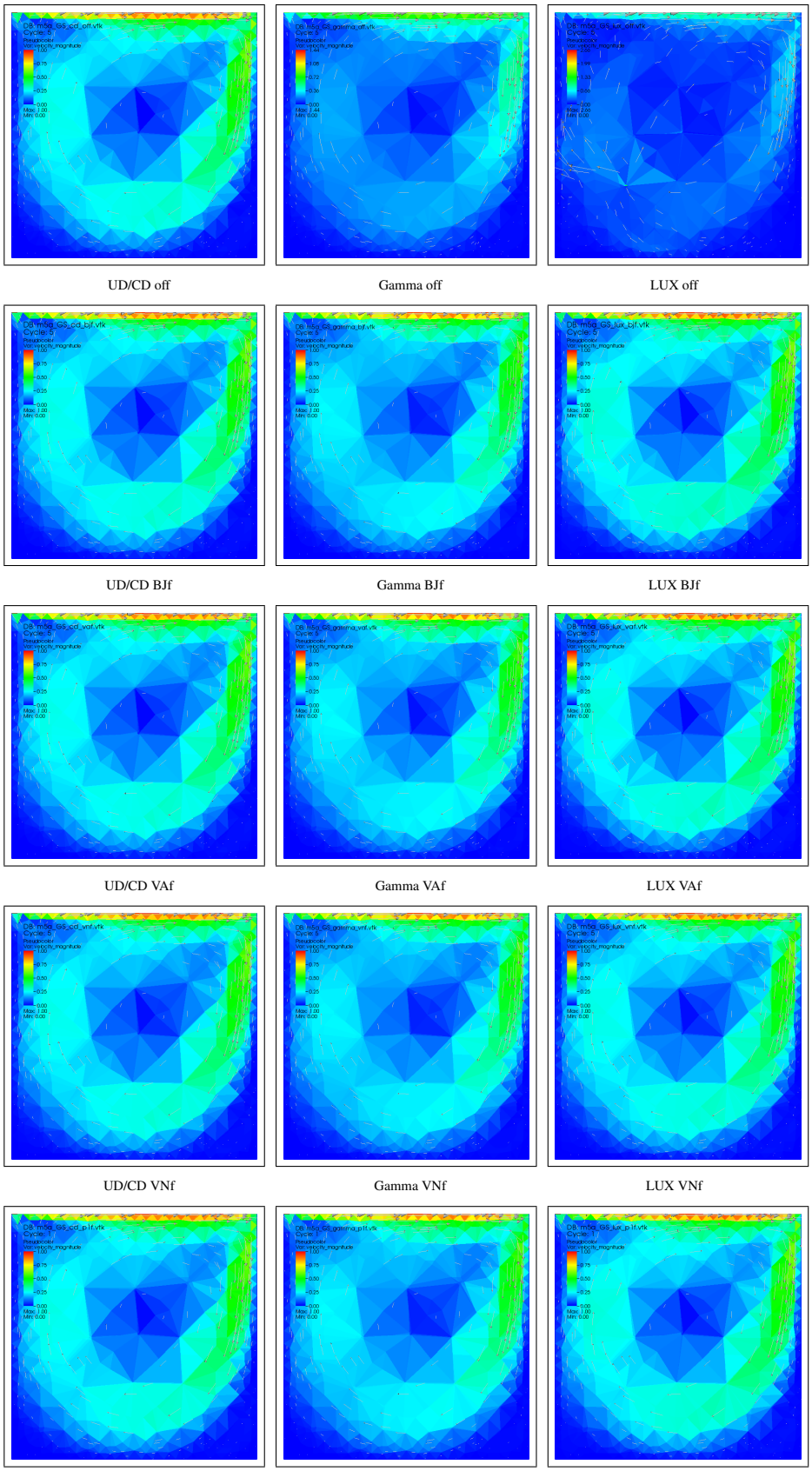


Figure 11.12: m5a LDC tet cases with Gauss and face based limiters

11.5 Plain flow from left to right

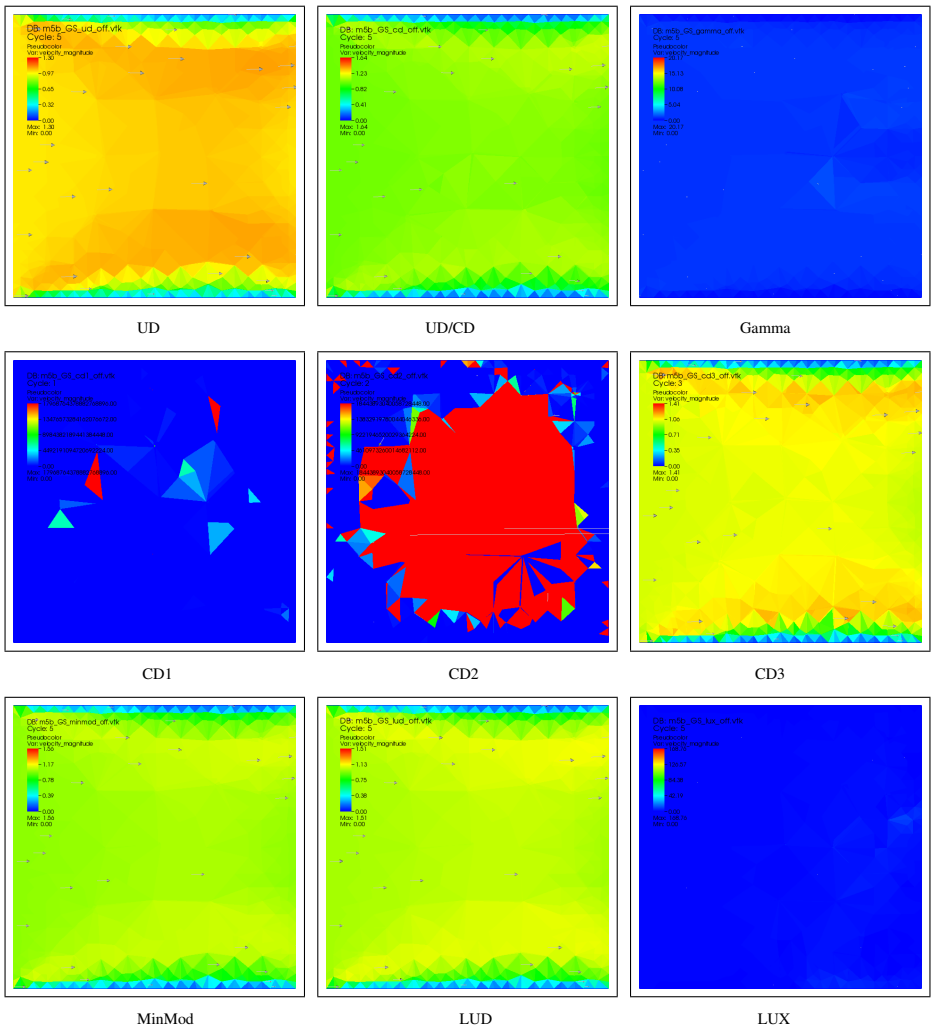


Figure 11.13: m5b plain flow tet cases with Gauss, no limiter

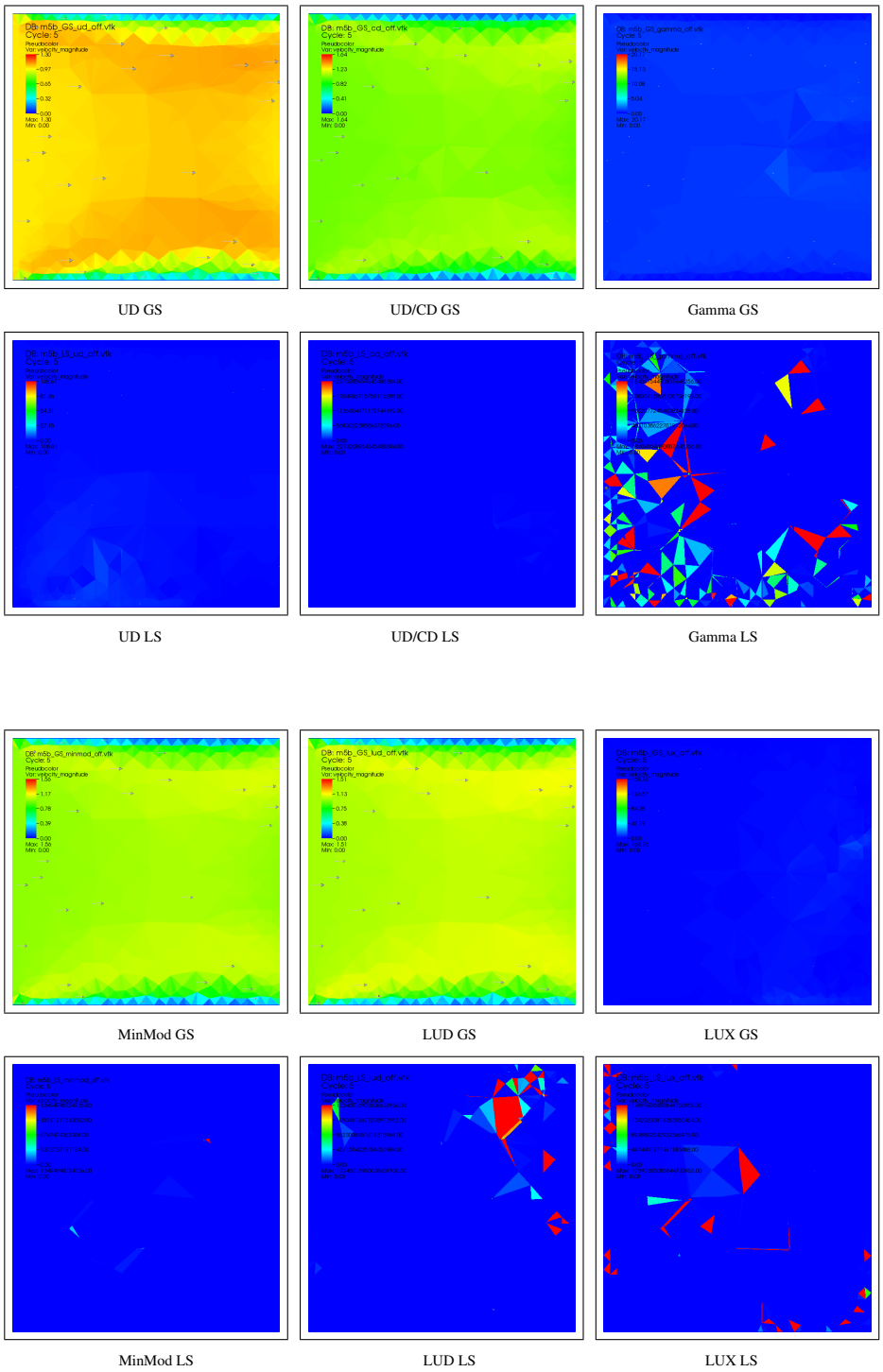


Figure 11.14: m5b plain flow tet cases with Gauss and Least Squares, no limiters used

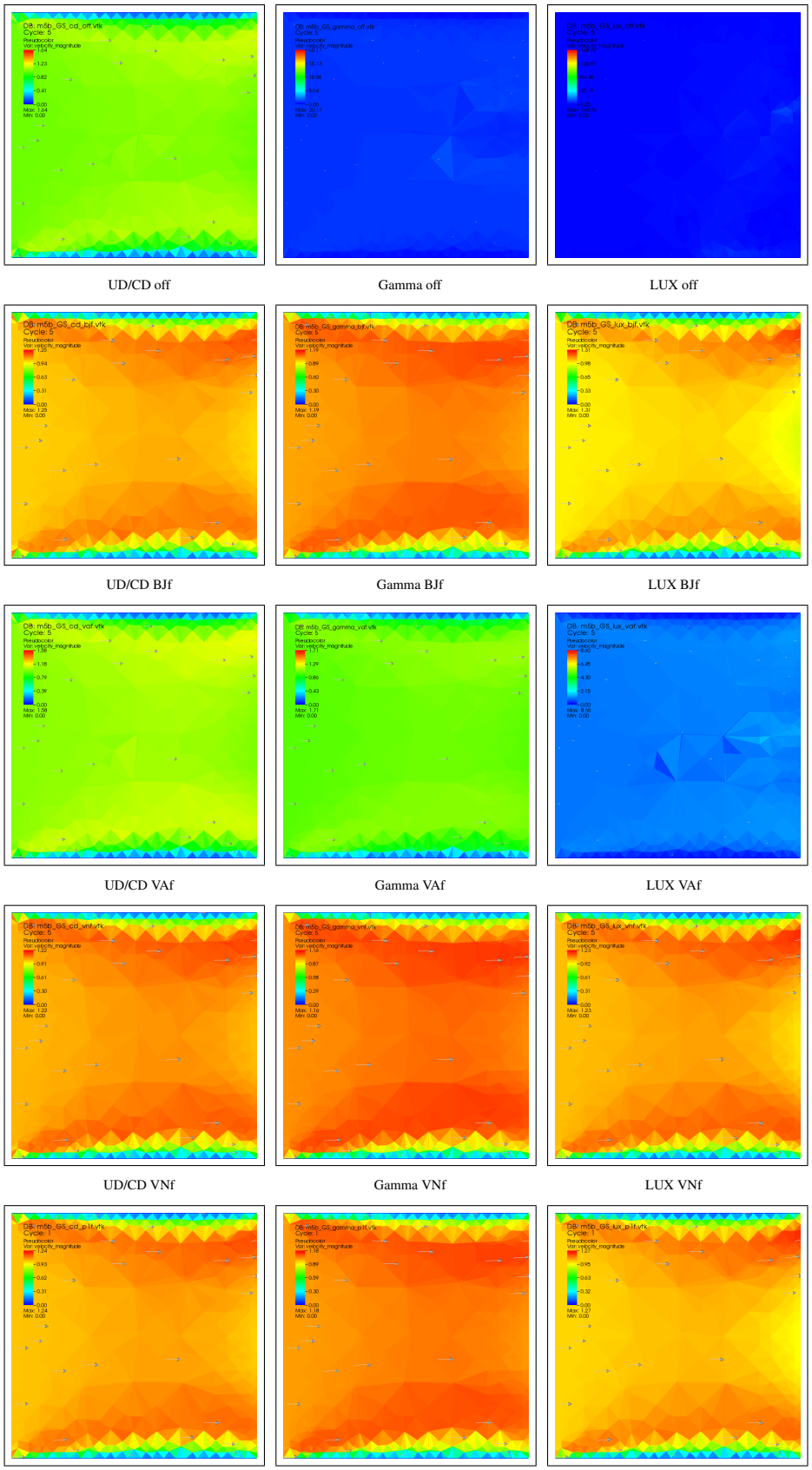


Figure 11.15: m5b plain flow cases with Gauss and face based limiters

11.6 Stagnation flow from top

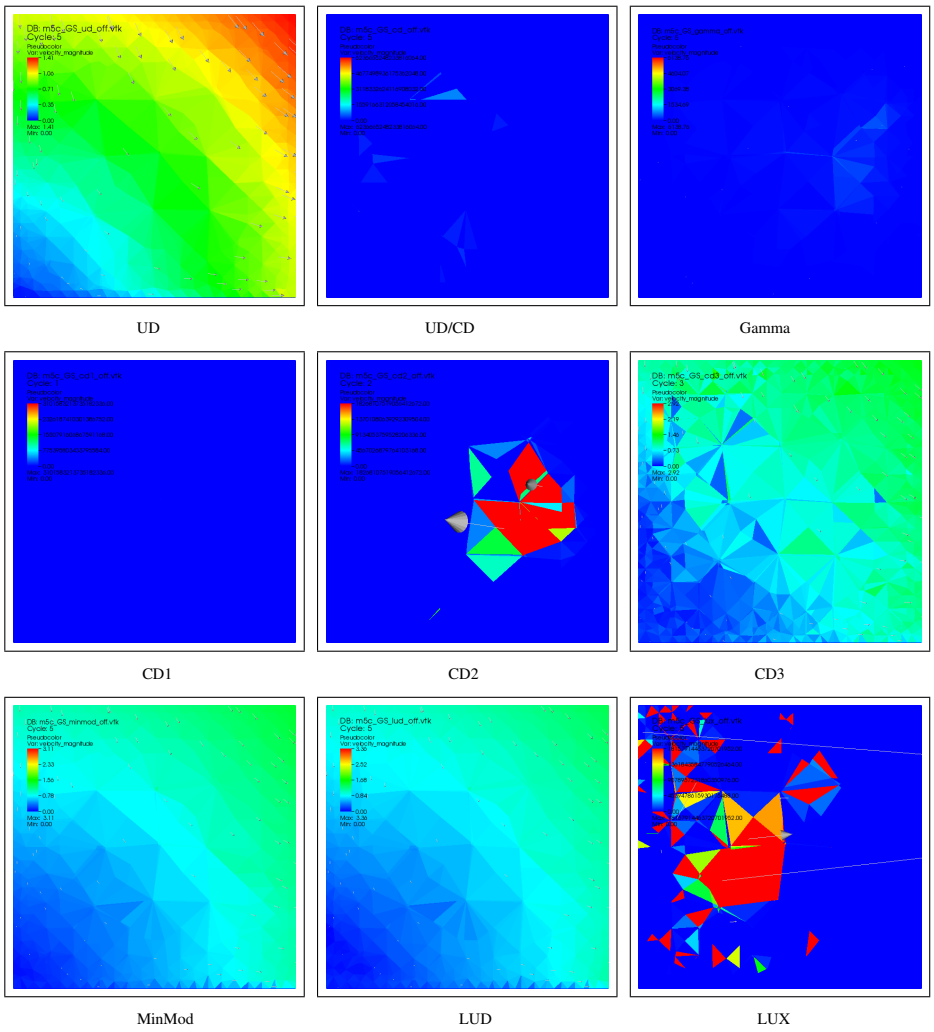


Figure 11.16: m5c stagnation flow tet cases with Gauss, no limiter

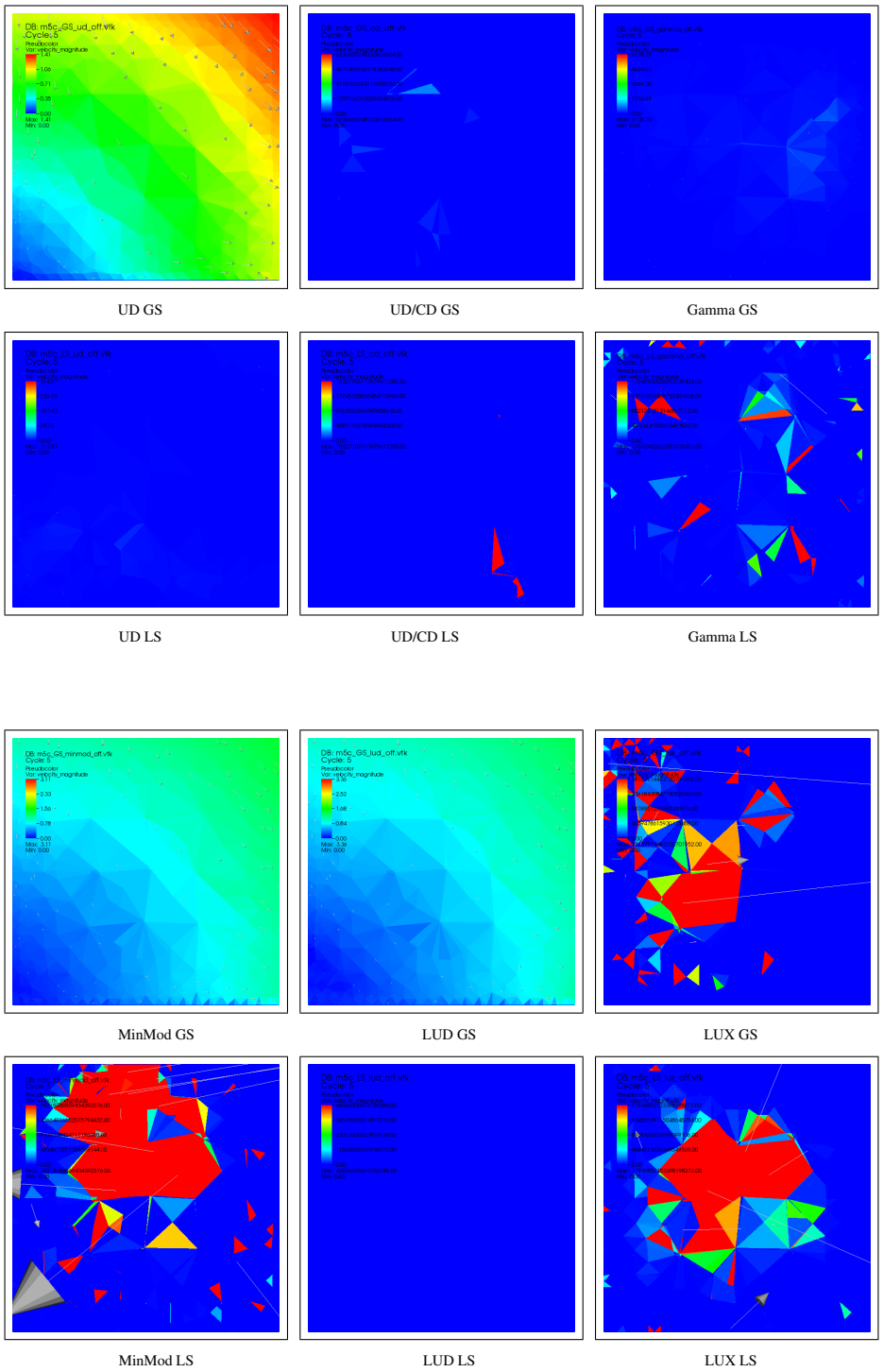


Figure 11.17: m5c stagnation flow tet cases with Gauss and Least Squares, no limiters used

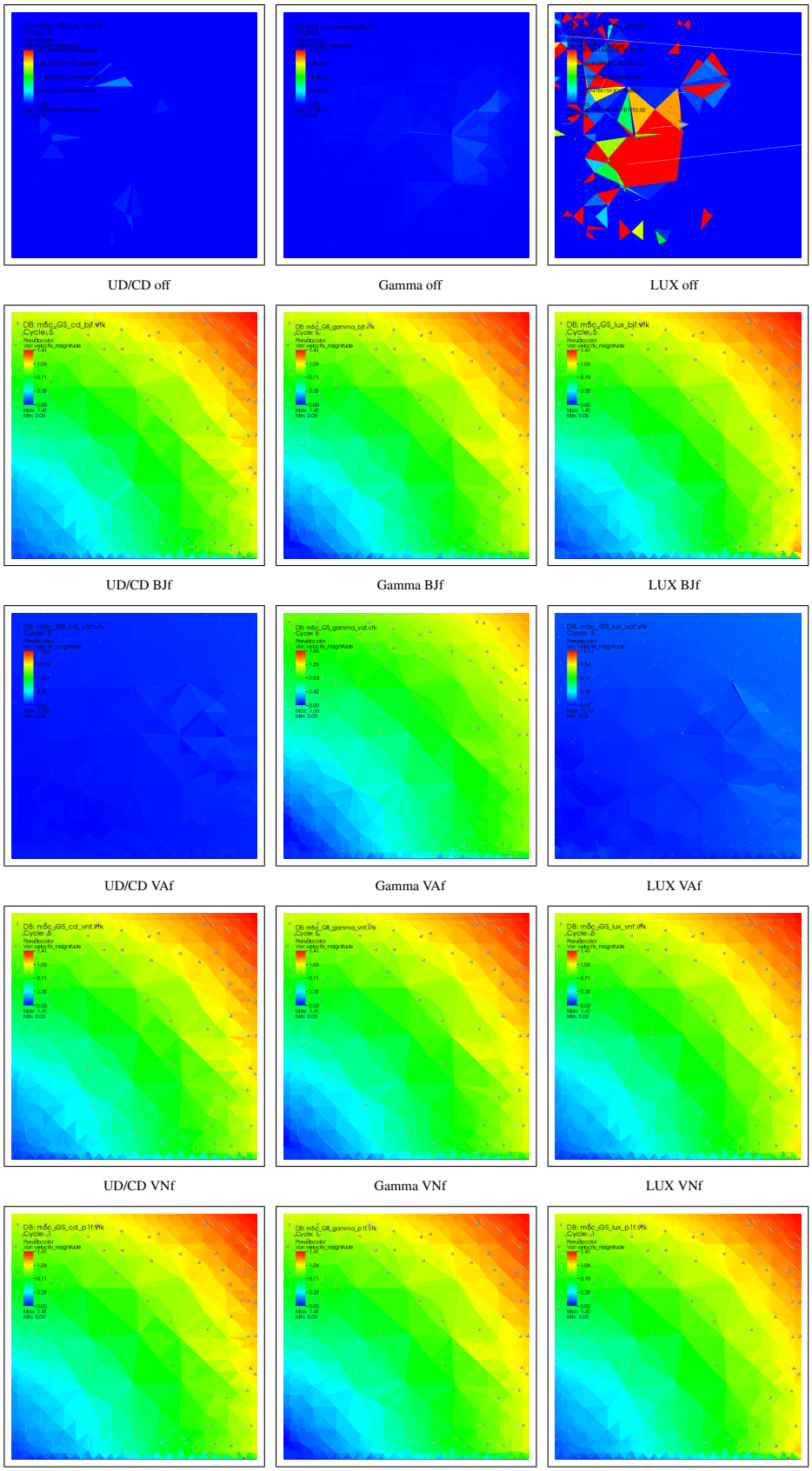


Figure 11.18: m5c stagnation flow cases with Gauss and face based limiters

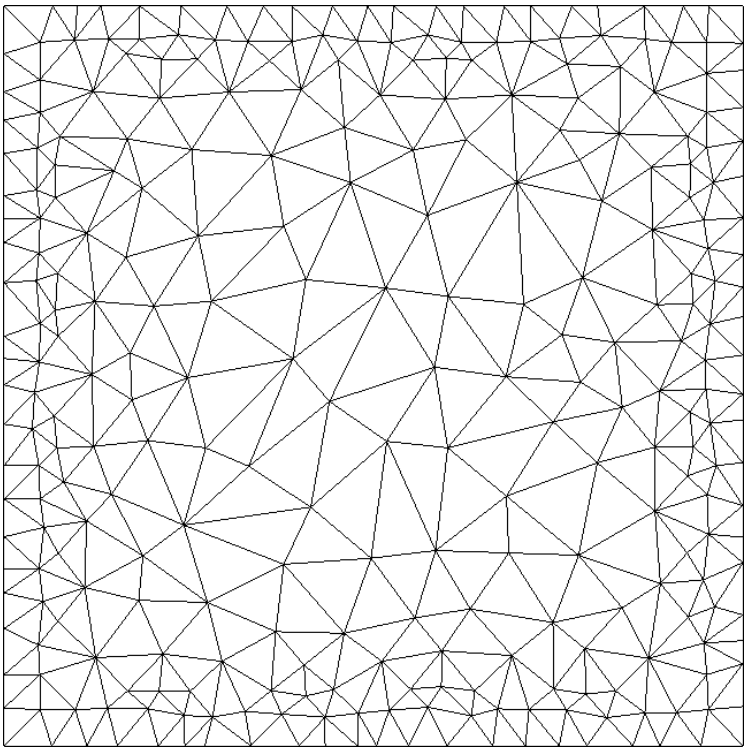


Figure 11.19: Mesh model m5

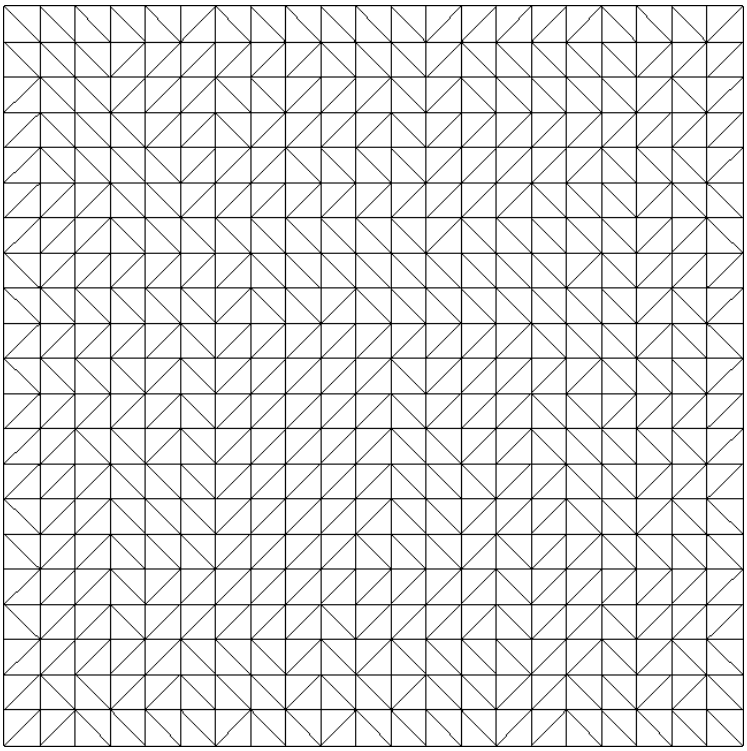


Figure 11.20: Mesh model m3