It can be used for applications ranging from obtaining good quality meshes without prior knowledge of the solution to adapting the meshes to follow transients and to iteratively adapt the finite element solution/mesh towards an optimal steady state. Examples of this are presented in Section 4 with application to unsteady fluid flow and steady-state radiation transport problems.

2. A mesh optimisation and adaptivity method

The mesh adaptivity approach described below is based on a variational functional which gauges the quality of the mesh including both element size and shape as objectives. The functional is optimised using local searches of the mesh connectivity and node positions. An appropriate definition of a metric based on the Hessian allows distances in the desired Euclidean space to be calculated, which in turn allows the element (length) size to be controlled by, for example, a specified interpolation error.

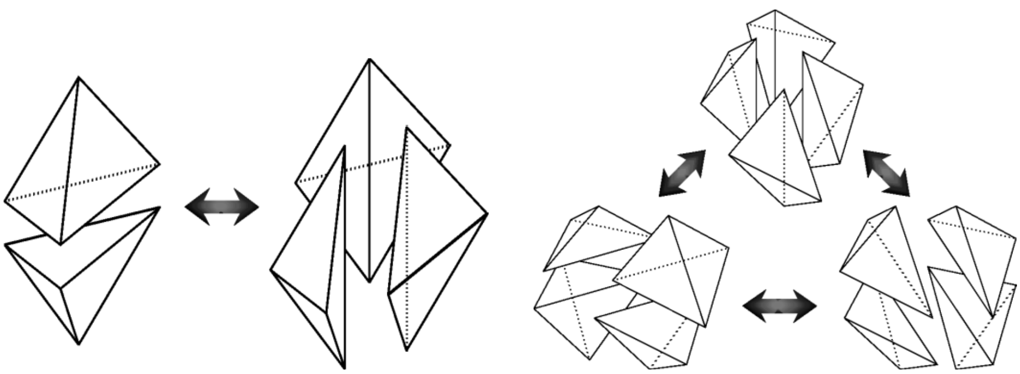
2.1. Optimisation heuristics

The optimisation method aims to improve, locally, the worst element. The overall objective function is the element quality associated with the worst element of the mesh. Within the optimisation, each element in the mesh is visited in turn and the following operations are performed within the vicinity of the element: (1) edge collapsing; (2) edge splitting; (3) face to edge (Fig. 1(b)) and edge to face (Fig. 1(a)) swapping; (4) edge swapping; (5) node movement.

The mesh optimiser proposes a new neighbourhood mesh configuration (local mesh transformation) which is a small change in the mesh, using one of the above operations. This is accepted as the current mesh configuration if the change in the maximum functional associated with all the elements affected by the change, is negative and less than a smallness parameter κ. In addition, if the maximum functional value of all the elements that would be affected by a local mesh transformation is less than a certain threshold value $\mathscr{F}_t$ (a value of 0.15 is used here) then this mesh transformation is not considered. The values of κ and $\mathscr{F}_t$ are defined bearing in mind the element quality, the most common of which (after mesh optimisation) is about 0.2 measured by Eq. (5), see Section 4. That is,

$$\max_{e'\in\mathscr{E}'}\{F_{e'}\} - \max_{e\in\mathscr{E}}\{F_e\} \leqslant -\kappa \quad \text{and} \quad \max_{e\in\mathscr{E}}\{F_e\} > \mathscr{F}_t \tag{1}$$

for elements in the set $\mathscr{E}$ and element functionals $F_e \; \forall e \in \mathscr{E}$ (which measure the quality of elements e) that will be effected by a proposed local mesh transformation; $\mathscr{E}'$ is the set of, and $F_{e'}$ are the functional values of the elements changed or created by the proposed local mesh transformation. A positive non-zero value of κ is used to avoid problems with round-off error and also provides some additional control over the CPU requirements of the algorithm ($\kappa = 0.01$ is the default and is used in the applications).



(a) Face to edge (b) Edge swapping

Fig. 1. Digram showing: (a) edge to face and face to edge swapping; (b) edge to edge swapping with four elements.