

# Solvelen

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Solución numérica de la ecuación de Lane-Emden de índice  $n$

solvelen versión 0.1.4  
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Este es el manual de solveln (versión 0.1.4, 16 May 2015), una solución numérica de la ecuación de Lane-Emden de índice  $n$ .

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# 1 Resumen

`solvelen` determina la solución numérica de la ecuación de Lane-Emden de índice politrópico  $n$ . En modo preciso, `solvelen` resuelve la ecuación en situaciones de interés físico, esto es, en casos esféricos con un contorno acotado. Tales situaciones se presentan cuando el índice politrópico toma valores entre  $-1 < n < 5$ .

`solvelen` es un aplicación informática que permite calcular la raíz, el valor numérico de la derivada en la raíz y otros valores frecuentemente asociados a las funciones de Lane-Emden de índice  $n$ .

Dada la importancia de la ecuación en astrofísica por modelar las estructuras estelares con razonable precisión, `solvelen` es una herramienta práctica que facilita y agiliza la evaluación numérica de algunos parámetros básicos que suceden en los problemas de estructura estelar.

## 2 Introducción

`solvelen` es una aplicación informática práctica desarrollada en lenguaje C++ que funciona mediante una interfaz de línea de comandos (CLI)<sup>1</sup> en una terminal para sistemas UNIX o en el símbolo de sistema para entornos operativos Windows (Woe)<sup>2</sup>.

La aplicación resuelve la ecuación de Lane-Emden de índice politrópico  $n$  en situaciones de caso esférico y contorno acotado ( $-1 < n < 5$ ) por medio del método de Runge-Kutta, ofreciendo diferentes resultados numéricos con sólo ingresar el valor numérico del índice politrópico. La aplicación ofrece los siguientes “servicios” o resultados numéricos:

- raíz de la función de Lane-Emden,  $X_o$ .
- valor numérico de la derivada de la función de Lane-Emden evaluada en la raíz,  $y'|_{X_o}$ .
- razón densidad media a densidad central  $(-3y'/x)|_{X_o}$ , masa adimensional  $(-x^2y')|_{X_o}$  y presión adimensional  $W_n$  [Cha39].
- archivo de datos en texto plano con la solución numérica de la función de Lane-Emden y su derivada desde  $x = 0$  hasta la raíz  $x = X_o$ .
- información del progreso de la solución numérica, apropiado cuando el índice politrópico es próximo a 5.

Los resultados numéricos son expresados con 12 dígitos de precisión para la raíz y la derivada, 8 dígitos de precisión para la razón de densidades, masa adimensional y presión adimensional, y 15 dígitos de precisión en el archivo de datos generado.

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<sup>1</sup> Acrónimo en inglés de Command Line Interface.

<sup>2</sup> Acrónimo en inglés de Windows Operating Environment.

### 3 La ecuación de Lane-Emden

La ecuación de Lane-Emden es una ecuación diferencial de segundo orden no-lineal, la cual describe la estructura de una esfera de gas politrópico que se mantiene en equilibrio bajo su propia gravedad, sujeto a las leyes de la termodinámica clásica; la consideración de un gas politrópico en el interior de la estructura se sigue a partir de los mecanismos de transporte de energía que suceden al interior de la estructura [MAB80].

Bajo tales consideraciones físicas, la ecuación de Lane-Emden se escribe como

$$y'' + 2y'/x + y^n = 0$$

siendo  $x$  proporcional a la distancia radial desde el centro de la estructura y  $y(x)$  proporcional a la densidad de la misma. Las condiciones de contorno para resolver la ecuación diferencial surgen de las circunstancias físicas del centro de la estructura donde la densidad alcanza su valor máximo, que en modo matemático se expresa como

$$y(0) = 1, \quad y'(0) = 0$$

Es bien conocido que la ecuación de Lane-Emden tiene soluciones analíticas con índice politrópico dado por 0, 1 y 5, mientras que para otros valores hay que recurrir a métodos numéricos o tablas de referencia [Hor86]. El método numérico usado en `solvelen` consiste en un método de Runge-Kutta de sexto orden para la solución de la ecuación diferencial y algoritmos de interpolación de Lagrange para el cálculo de la raíz de la función.

Con el método de Runge-Kutta usado es posible obtener una buena precisión con bajo gasto computacional frente a otros métodos numéricos [Lut68]. De acuerdo con lo anterior, se usa el método con un tamaño de paso fijo  $h$  de 0.00025 entre  $x=0$  y el primer cero de la función; comparando las soluciones numérica y analítica para el caso  $n=1$ , el error obtenido es del orden de  $10^{-14}$  con unas 12000 iteraciones aproximadamente.

Los algoritmos de interpolación usados en `solvelen` consisten en la realización de una extrapolación y luego de una interpolación con los últimos puntos obtenidos en la solución numérica; en ambas situaciones se aplican interpolaciones de Lagrange de 4 puntos con abscisas igualmente espaciadas [AS72].

En la extrapolación de la función de Lane-Emden  $y(x)$ , el último punto  $y_N = y(x_N)$  es un punto falso hallado mediante la interpolación de 4 puntos separados por una longitud  $2h$ , asumiendo que  $y_N < 0$ ; cuando el índice  $n$  es un número entero,  $y_N$  es un valor real aproximado de la función exacta, pero cuando  $n$  es un número real no entero,  $y_N$  es un valor real ficticio o falso de la función exacta, puesto que la evaluación de  $y^n$  en la ecuación diferencial con  $y < 0$  arroja un número complejo.

Seguido de la extrapolación, se continúa con una interpolación propiamente dicha de la función de Lane-Emden en el intervalo  $[x_{N-1}, x_N]$  con el fin de hallar la raíz  $y(x) = 0$  a través del método de la bisección. Para ello, se usan 4 puntos separados por una longitud  $h$ , asumiendo que  $y(x_{N-1})$  y  $y(x_N)$  son de signos opuestos, lo cual asegura la existencia de una raíz dentro del intervalo.

Hallada la solución numérica de la ecuación diferencial y la raíz de la función de Lane-Emden, `solvelen` muestra a través de la pantalla (CLI) los resultados hallados según las ordenes ingresadas por el cliente.

## 4 Usando el programa

A continuación se explica el funcionamiento de `solvelen` e información acerca de los mensajes de error emitidos por el programa.

### 4.1 Invocando el programa

El programa “corre” a través de la línea de comandos (CLI). El modo de ejecución o de uso es:

```
solvelen [opciones] <índice politrópico>
```

El argumento [**opciones**] hace referencia a la invocación de uno de los servicios que brinda el programa; cualquier opción puede redactarse en modo corto o en modo largo: una opción corta inicia con ‘-’ y una letra representativa del servicio, mientras que una opción larga inicia con ‘--’ y una palabra representativa del servicio. El argumento **<índice politrópico>** hace referencia al valor numérico del índice politrópico  $n$ ; dicho número debe estar en formato decimal y con un valor de  $-1 < n < 5$  que son precisamente los valores en donde la función de Lane-Emden tiene una raíz. Valores de  $n$  fuera del rango mencionado no son evaluados por `solvelen` puesto que no existe raíz en el rango de la función. Como ejemplo, para hallar la raíz de la función de Lane-Emden con índice  $n=1.509$ , en la línea de comandos se escribe

```
solvelen -r 1.509
3.66443777776
```

o en modo de opción larga

```
solvelen --raiz 1.509
3.66443777776
```

En modo similar funciona la evaluación de la derivada de la función de Lane-Emden; `solvelen` evalúa la derivada de función en la raíz mediante la opción `-d` o `--derivada`. Si el índice es negativo, `solvelen` muestra el mensaje:

```
Advertencia: ...; índice negativo, resultado con baja exactitud.
```

La razón de dicho mensaje es debida a que el método de Runge-Kutta disminuye la convergencia de la solución a causa del factor  $y^n$ , el cual necesita de un paso de integración  $h$  inferior al considerado para lograr una buena exactitud frente a los valores encontrados en la literatura.

La opción `-a` o `--all` permite hallar algunos parámetros de uso frecuente en la función de Lane-Emden, ellos son: razón densidad media a densidad central, masa adimensional y presión adimensional. Luego de procesar la opción con el índice requerido, `solvelen` muestra en una línea el valor numérico de los anteriores parámetros junto con la raíz y la derivada de la función evaluada en la raíz; el programa presenta los resultados en el modo siguiente:

n	Xo	y'   Xo	(-3y' / x)   Xo	(-x^2y')   Xo	Wn
índice	raíz	derivada	razón densidad	masa	presión
politrópico			media/central	adimensional	adimensional

cada resultado es expresado en notación científica de la forma  $Aeb$  que corresponde al número  $A \times 10^b$ . Como ejemplo, para el índice  $n=1.509$ , `solvelen` muestra

```

solvelen --all 1.509
      n      Xo      y'|Xo (-3y'/x)|Xo (-x^2y')|Xo      Wn
1.509 3.6644e+00 -2.0164e-01 1.6508e-01 2.7077e+00 7.8005e-01

```

La opción `-o` o `--outfile` permite generar un archivo de datos con la solución numérica de la función de Lane-Emden para un índice dado. Esta opción requiere como argumento adicional y obligatorio, el nombre de un archivo de salida donde se guardaran los datos. En dicho archivo, queda registrado en columnas el valor de  $x$ , valor de la función  $y(x)$  y la derivada  $y'(x)$  desde cero hasta la raíz de la función; los valores de  $x$  se incrementan con un tamaño de paso de 0.001. Para valores del índice próximos a 5 se recomienda especial atención en el archivo generado, ya que la raíz de la función tiende a infinito y esto crearía un sobre-flujo de datos con un archivo de gran tamaño. Para un ejemplo con índice  $n=1.509$  guardado en el archivo `datos.txt`, en la línea de comandos se escribe

```
solvelen --outfile datos.txt 1.509
```

Una forma rápida de visualizar los datos en una gráfica es por medio del programa `gnuplot` usando la orden

```
plot './datos.txt' using 1:2 with lines
```

`solvelen` tiene una opción de modo detallado que permite ver el progreso de la solución del método de Runge-Kutta. Conforme el índice se acerca a 5, el primer cero de la función de Lane-Emden tiende hacia infinito, lo cual requiere demasiadas iteraciones para alcanzar el valor negativo de la función, haciendo del proceso de búsqueda de la raíz dispendioso en tiempo. Para apreciar el progreso de la iteraciones transcurridas en dichas situaciones, la opción `-v` o `--verbose` muestra el avance porcentual del progreso de búsqueda del cero; la opción puede combinarse con cualquiera de las anteriores opciones y en cualquier orden siempre y cuando el índice sea el último argumento ingresado. Si el tamaño de paso  $h$  es modificado en el código fuente de la aplicación, la opción de modo detallado pierde su consistencia, puesto que la función que determina el número total de iteraciones está adecuada para un tamaño de paso  $h=0.00025$ .

Los siguientes ejemplos muestran el uso de `--verbose`

```

solvelen -r -v 1.509
5 %
10 %
...
95 %
100 %
3.6644377776

solvelen --verbose --outfile dat 1.509
5 %
...
100 %

```

Finalmente, las opciones `--version` y `--help` muestran información relevante del programa. Con `--version` se obtiene la versión y licencia del programa, mientras que `--help` muestra una ayuda informativa breve mencionando las opciones del programa.

## 4.2 Diagnóstico de errores

Durante la ejecución del programa pueden presentarse algunos errores, causando la terminación inmediata del programa. Cuando se produce un error, el programa muestra en la salida estándar un mensaje de error en la forma

Error: ‘<causa posible>’; ‘<descripción>’.

En lo que sigue se explican los mensajes de error del programa, teniendo en cuenta que su origen puede deberse a la redacción de argumentos y/o opciones; la discusión no profundiza en errores de compilación o de ejecución sobre un sistema operativo.

‘<causa posible> - <descripción>’

Explicación

‘argumento nulo - no existen opciones’

El programa no ha recibido algún argumento.

‘xxx - opción desconocida’

La opción ‘xxx’ no hace parte del conjunto de opciones del programa.

‘argumento nulo - no existe argumento’

El programa ha recibido una opción pero no se ha ingresado el índice como argumento.

‘xxx - demasiados argumentos’

El programa ha recibido más argumentos de los necesarios; el programa generalmente recibe 2 argumentos o 3 argumentos cuando se incluye la opción `--verbose`. La opción `--outfile` necesita 3 argumentos o 4 argumentos cuando se incluye el modo `--verbose`.

‘-v - demasiados ‘verbose’

El programa ha recibido más de una vez la opción `--verbose`.

‘xxx - NaN, índice politrópico no es un número’

NaN es el acrónimo en inglés de “Not a Number”, y por tanto, el índice politrópico ‘xxx’ ingresado no es un número en formato decimal.

‘xxx - índice politrópico debe ser  $-1 < n < 5$ ’

El índice politrópico se encuentra en el rango donde la función de Lane-Emden no tiene raíz.

‘método bisección - la función no pasa por el eje x en el intervalo [a, b]’

Error en el método de la bisección; puede deberse a que la función de Lane-Emden es positiva en el intervalo  $[a, b]$ .



## 5 Resultados numéricos

En la tabla se muestra una comparación numérica del primer cero de la función de Lane-Emden obtenido por `solvelen`, el valor exacto reportado en la referencia [Hor04] con 9 dígitos y el error porcentual.

$n$	<code>solvelen</code>	Valor exacto	Error % (*)
-0.9	2.05040657666	2.05040073	2900
-0.2	2.34663986995	2.34663985	8.5
0	2.44948974278	2.44948974	1.1
1	3.14159265359	3.14159265	1.1
2	4.35287459594	4.35287460	0.93
3	6.89684861937	6.89684862	0.091
4	14.9715463488	14.9715463	3.3
4.5	31.8364632445	31.8364632	1.4
4.99	1758.18915649	1758.18915	3.7

(\*) El error porcentual debe multiplicarse por  $10^{-7}$ .

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